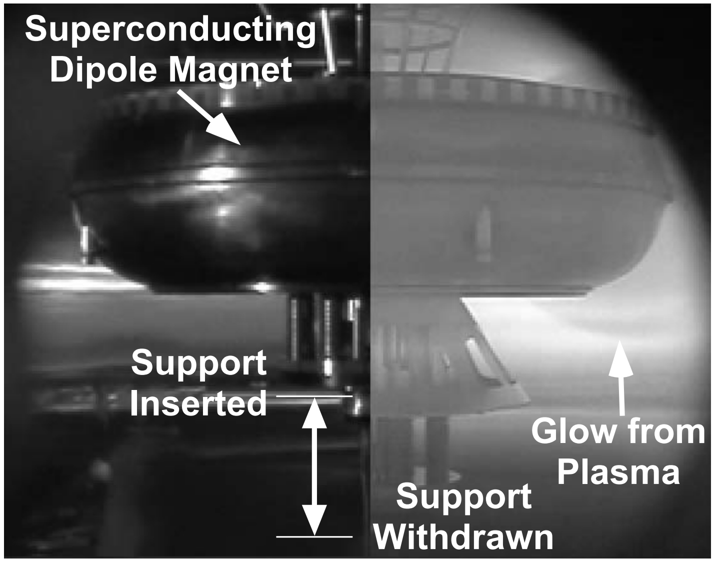

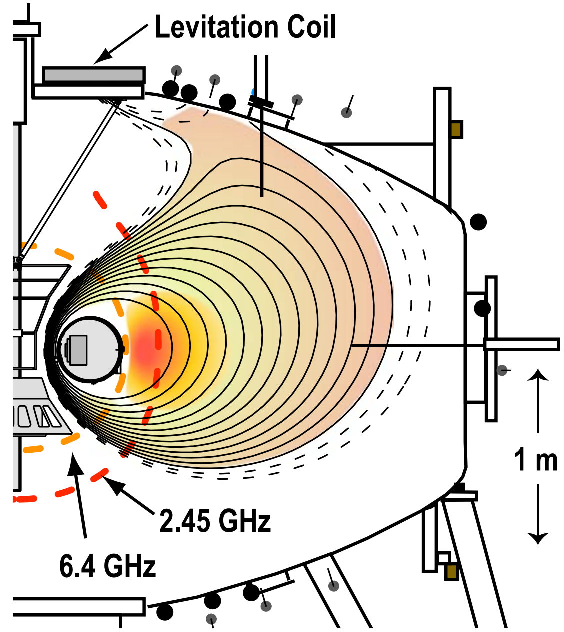

Investigation of the Variation of Measured Particle Diffusion Coefficient

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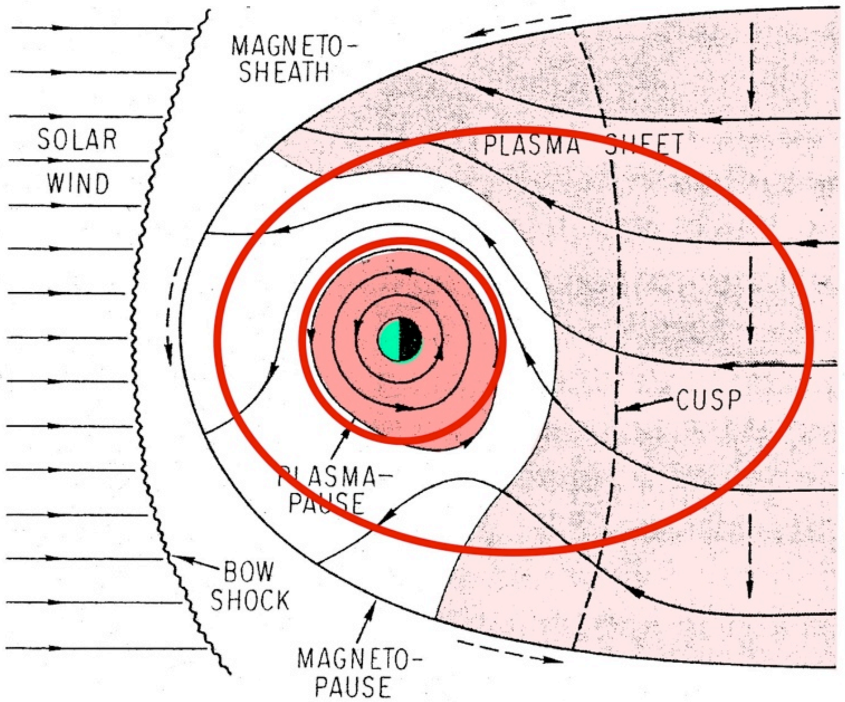
Abstract

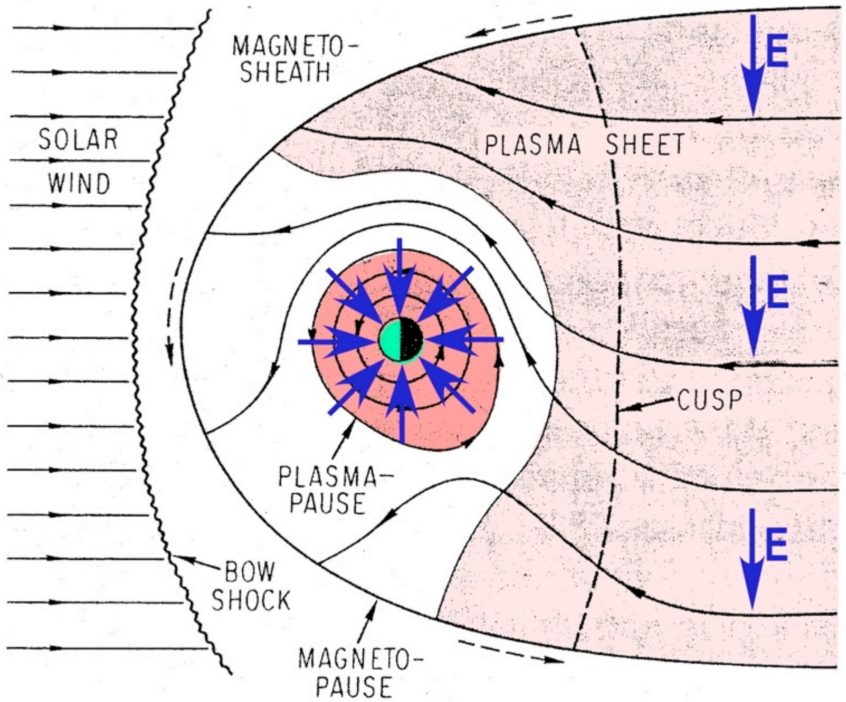
Measurements of the time evolution of the plasma density profile with a levitated dipole in LDX determine the radial particle diffusivity, provided the ionization source is known. In discharges where the particle ionization source appears to be at the outer plasma edge, we observe an anomolous inward particle pinch leading to centrally peaked plasma profiles. The observed inward pinch corresponds to a particle diffusivity that is independent of or varies weakly on radius. In these circumstances, the magnitude of the diffusion coefficient is equal to the value estimated from the turbulent electric field fluctuations measured at the edge with an array of floating potential probes, or $D \approx R^2 \langle E_\phi^2 \rangle \tau_c$, where τ_c is the correlation time. The fluctuation level varies as the plasma density, gas fueling, and microwave heating power changes. We discuss the relationship between fluctuations and particle transport and describe the relationship between particle diffusivity and fluctuation level for several conditions.

Particle Diffusion Due to Low Frequency Fluctuations

- In strongly magnetized plasma, non-axisymmetric fluctuations “break” the third adiabatic invariant while preserving μ and J.
- With random fluctuations, radial diffusion results, driving the particle distribution to the stationary condition, $\partial F/\partial \psi \rightarrow 0$, and an inward turbulent pinch.
- Magnetic fluctuations: Nakada and Mead, *JGR*, 1965
- Electric fluctuations: T. Birmingham, *JGR*, 1969
- (μ, J) conservation: Warren, Bhattacharjee, Mauel, *GRL*, 1993

Perturbed ψ Caused by Global Fluctuations of Geomagnetic Cavity (Easily Measured!)





$$\delta A_\phi \sim \overbrace{\frac{L}{4}\left(\frac{R_e}{R_m}\right)^3}^{\text{Axisymmetric}} \underbrace{- \frac{4}{30}\frac{L^2}{R_e}\left(\frac{R_e}{R_m}\right)^4 \cos\phi}_{m=\pm 1} + \ldots$$

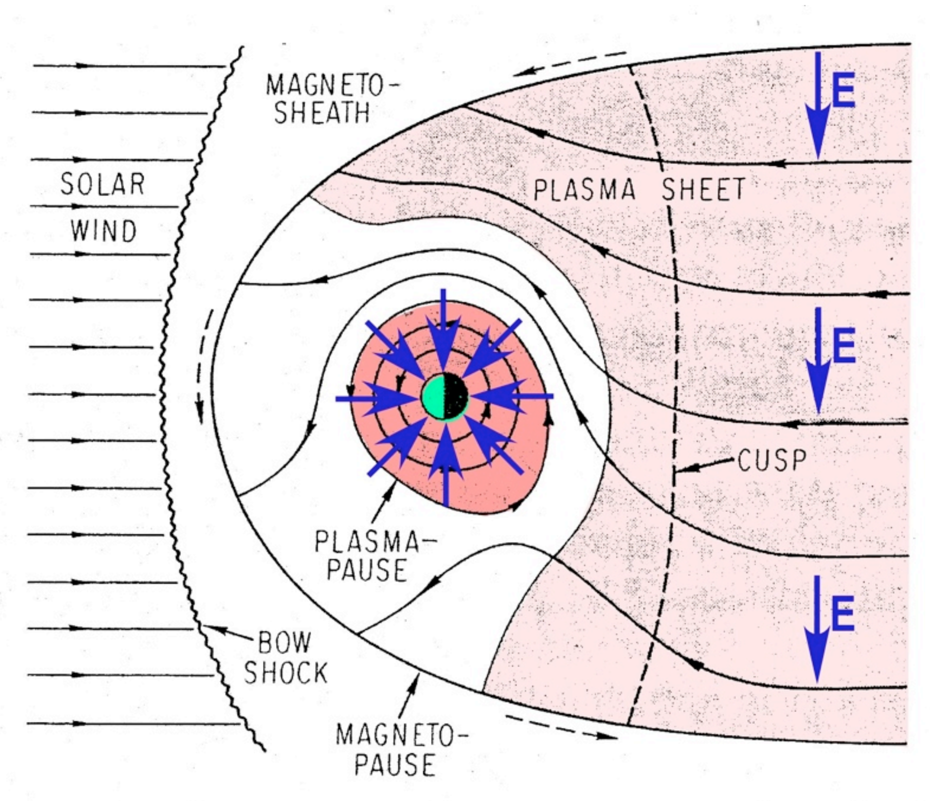
$$\delta \Phi \sim - \overbrace{E_c\left(\frac{R_e^2}{L}\right)}^{\text{Axisymmetric}} + \underbrace{E_c L \sin\phi}_{m=\pm 1} + \ldots$$

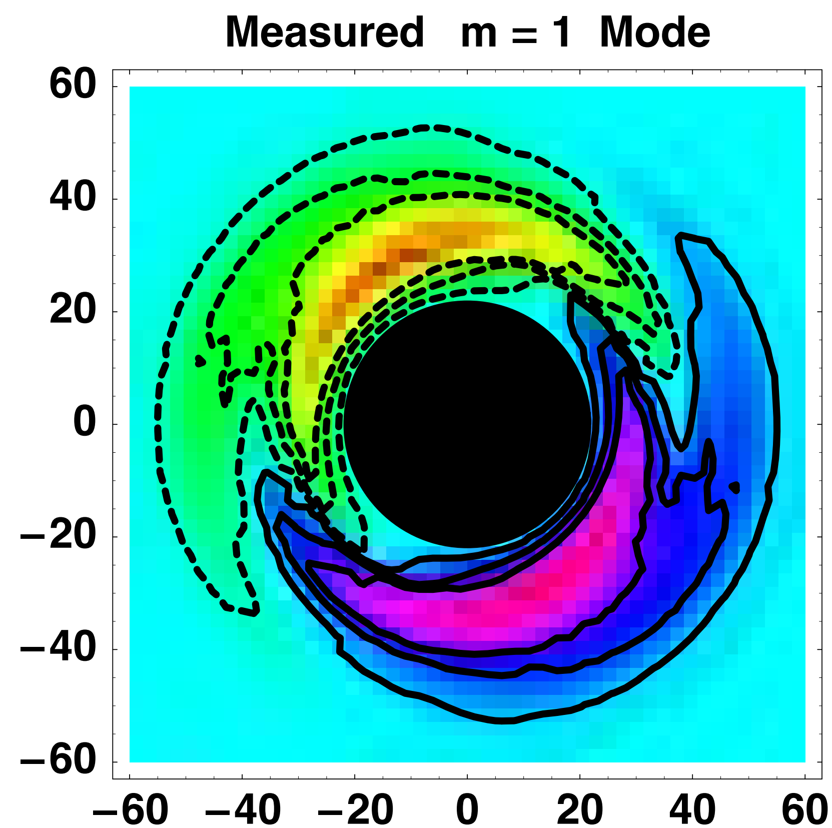
Nakada and Mead, *JGR* (1965)

T. Birmingham, *JGR* (1969)

Diffusion Coefficient Depends upon Random and/or Turbulent **E**×**B** Spectrum

(and ***not on the structure*** of the convection field!)





Open Streamlines:
Magnetosphere

Closed Streamlines:
Laboratory Dipole

Low-Frequency Dynamics is One-Dimensional

(1D, $k_\perp \rho \ll 1$, Gyrokinetics!)

$$\mathcal{H} = \frac{m_e c}{2e} \rho_{||}^2 B_0^2 + \mu \frac{c}{e}(B_0 + \delta B) - c\delta\Phi$$

$$B_0 \gg \delta B\,,\ \delta\mathbf{B} = \nabla \times \delta\mathbf{A}\,,\ \text{and } \delta\mathbf{E} = -\nabla\delta\Phi - \frac{1}{c}\frac{\partial \delta\mathbf{A}}{\partial t}$$

2D Phase Space

$$\left\{\begin{aligned} \dot{\mu} &= 0 \ (\omega \ll \omega_c) \\ \dot{J} &= 0 \ (\omega \ll \omega_b) \\ \dot{\psi} &= -\frac{\partial \mathcal{H}}{\partial \phi} = -c\frac{\partial \delta\Phi}{\partial \phi} + L\frac{\partial \delta A_\phi}{\partial t} \ (\propto \delta\mathbf{E} \times \mathbf{B}) \\ \dot{\phi} &\approx \mu \frac{c}{e}\frac{\partial B}{\partial \psi} - c\frac{\partial(\Phi + \delta\Phi)}{\partial \psi} \approx \omega_d(\mu, \psi) + \omega_E(\psi) \end{aligned}\right.$$

A. Chan, L. Chen, R. White, *GRL* (1989)

Adiabatic Radial Dynamics

$$F(\mu,\, J,\, \psi,\, \phi,\, t) = F_0(\mu,\, J,\, \psi) + \delta F(t)$$

$$\frac{\partial F}{\partial t} + \dot{\phi}\frac{\partial F}{\partial \phi} + \dot{\psi}\frac{\partial F}{\partial \psi} = 0$$

$$\left(\frac{\partial}{\partial t} + (\omega_D + \omega_E)\frac{\partial}{\partial \phi}\right)\delta F + \dot{\psi}\frac{\partial F_0}{\partial \psi} \approx 0$$

Linear Response...

$$\delta F(t) - \delta F(0) \approx -\int_0^t dt' \dot{\psi}(t')\frac{\partial F_0}{\partial \psi}$$

Gyrokinetic Quasilinear Diffusion...

$$\frac{\partial F_0}{\partial t} = \frac{\partial}{\partial \psi} D_{\psi\psi}\frac{\partial F_0}{\partial \psi} \text{ with } D_{\psi\psi} = \underbrace{\int_0^t}_{\text{Correlation along Particle Orbit}} dt' \dot{\psi}(t')\dot{\psi}(0) = \tau_{cor}\underbrace{\left|\frac{\partial \Phi}{\partial \varphi}\right|^2}_{\text{Fluctuating } E \times B}$$

T. Birmingham, *JGR* (1969)

MHD Radial Diffusion

If the correlation time is independent of particle drift velocity, then low-frequency gyrokinetics and MHD are equivalent…

$$N \equiv \int \frac{ds}{B} n = \delta V \langle n \rangle \approx N_0(\psi) + \delta N \,, \; \mathbf{E} \cdot \mathbf{B} = 0 \,, \text{and } \mathbf{E} = -\nabla \Phi$$

$$\frac{\partial N}{\partial t} - \frac{\partial}{\partial \varphi} \left(N \frac{\partial \Phi}{\partial \psi} \right) + \frac{\partial}{\partial \psi} \left(N \frac{\partial \Phi}{\partial \varphi} \right) = S$$

Linear MHD Response…

$$\delta N(t) - \delta N(0) \approx - \oint_0^t dt' \dot{\psi}(t') \frac{\partial N_0}{\partial \psi}$$

MHD Quasilinear Diffusion…

$$\frac{\partial N_0}{\partial t} = \langle S \rangle + \frac{\partial}{\partial \psi} D_{\psi\psi} \frac{\partial N_0}{\partial \psi} \text{ with } D_{\psi\psi} = \underbrace{\oint_0^t dt' \dot{\psi}(t') \dot{\psi}(0)}_{\text{Correlation along Flux-tube Motion}} = \tau_{cor} \underbrace{\left| \frac{\partial \Phi}{\partial \varphi} \right|^2}_{\text{Fluctuating } E \times B}$$

Plasma/Particle ExB Motion

$$\mathbf{V} = -\hat{\varphi} R \frac{\partial \Phi}{\partial \psi} + \frac{\hat{\psi}}{RB} \frac{\partial \Phi}{\partial \varphi}.$$

$$\dot{\psi} = \nabla \psi \cdot \mathbf{V} = \frac{\partial \Phi}{\partial \varphi} = -R E_{\varphi}$$

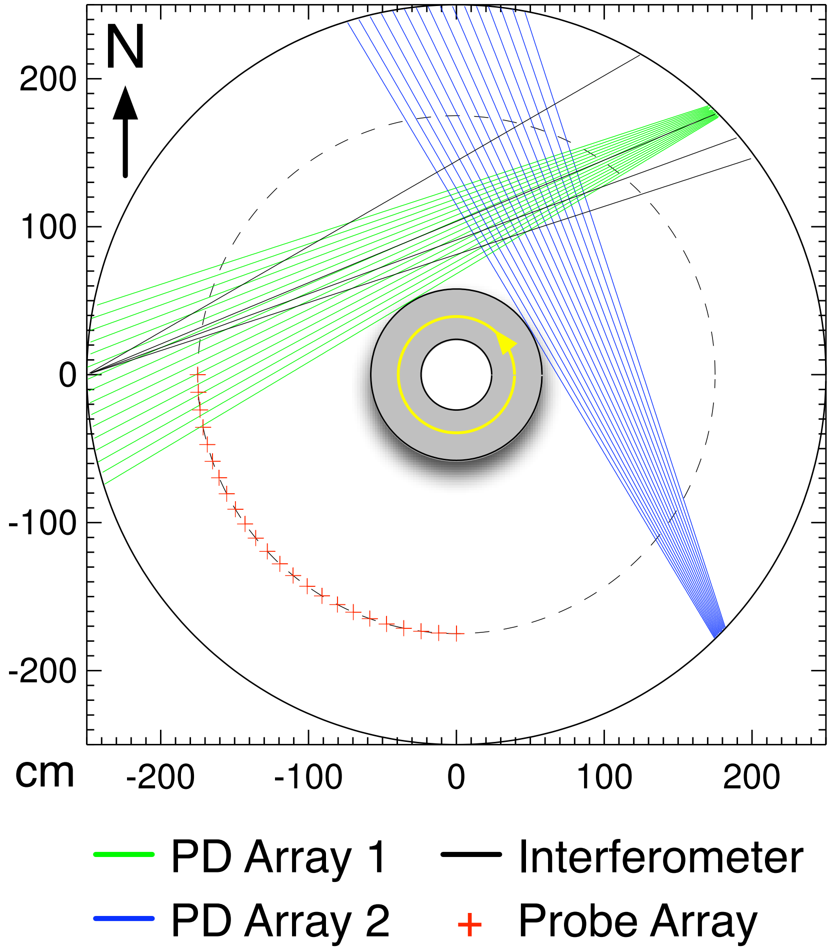
$$D = R^2 \langle E_{\varphi}^2 \rangle \tau_c$$

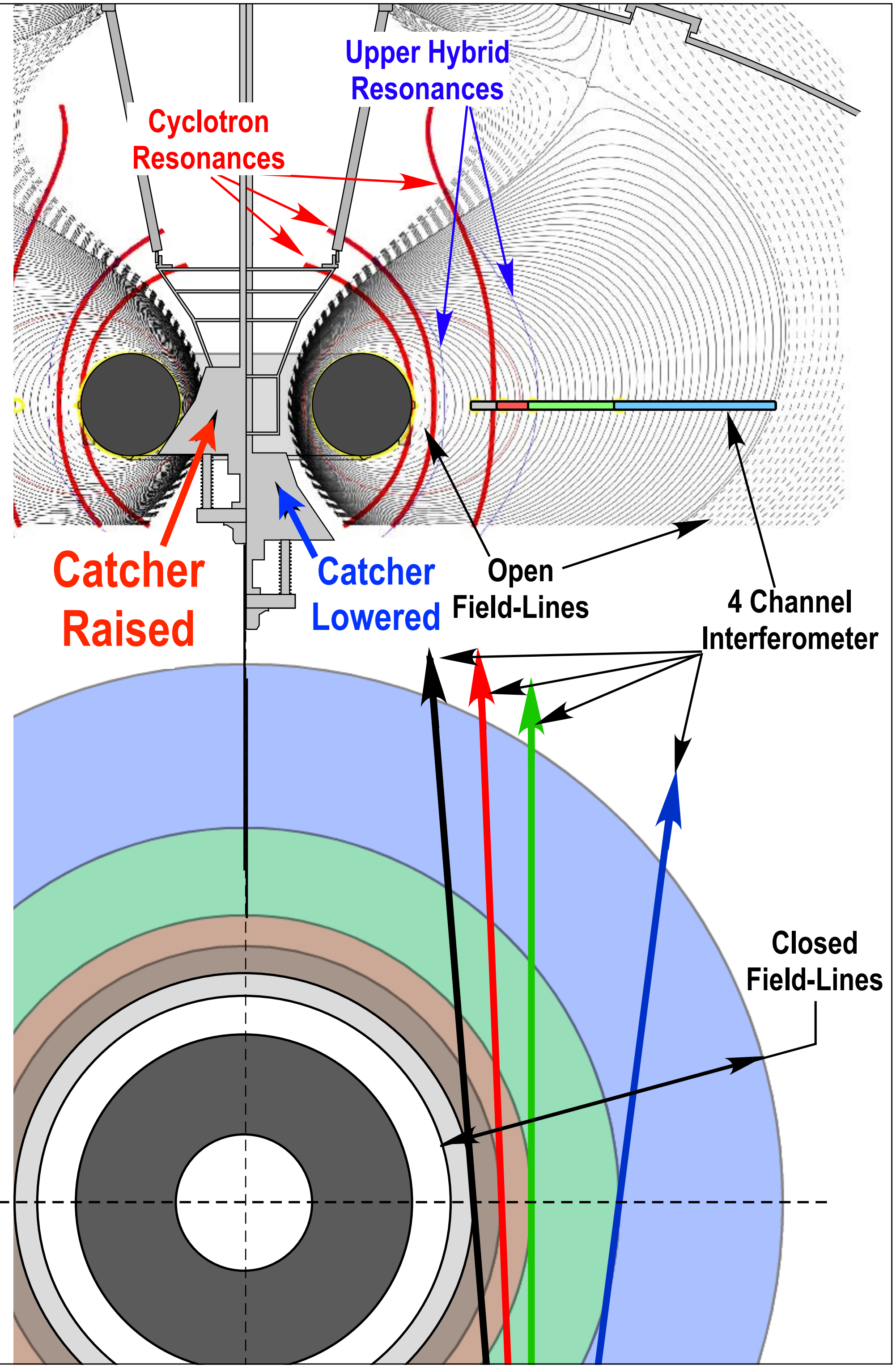
Flux-tube and particle diffusion are equal whenever τ_{cor} is independent of particle energy.

Measuring Radial Diffusion

- What is the RMS level of $(RE_\phi)^2$?
  Answer: RE_ϕ measured directly with edge probe array.
- What is the correlation time, τ_{cor}?
  Answer: Quasi-steady state discharges in LDX provide *long-time* data records for converged statistics.
- Does density profile evolve in accordance to random **E×B** diffusion?
  Answer: Yes! (In certain cases) line-density measurements show inward turbulent particle pinch and quasilinear profile relaxation.
- What is the radial profile of $D_{\psi\psi}$?
  Answer: Profile of light emission show fluctuations exist throughout plasma, consistent with turbulent particle pinch.

LDX Diagnostic Views



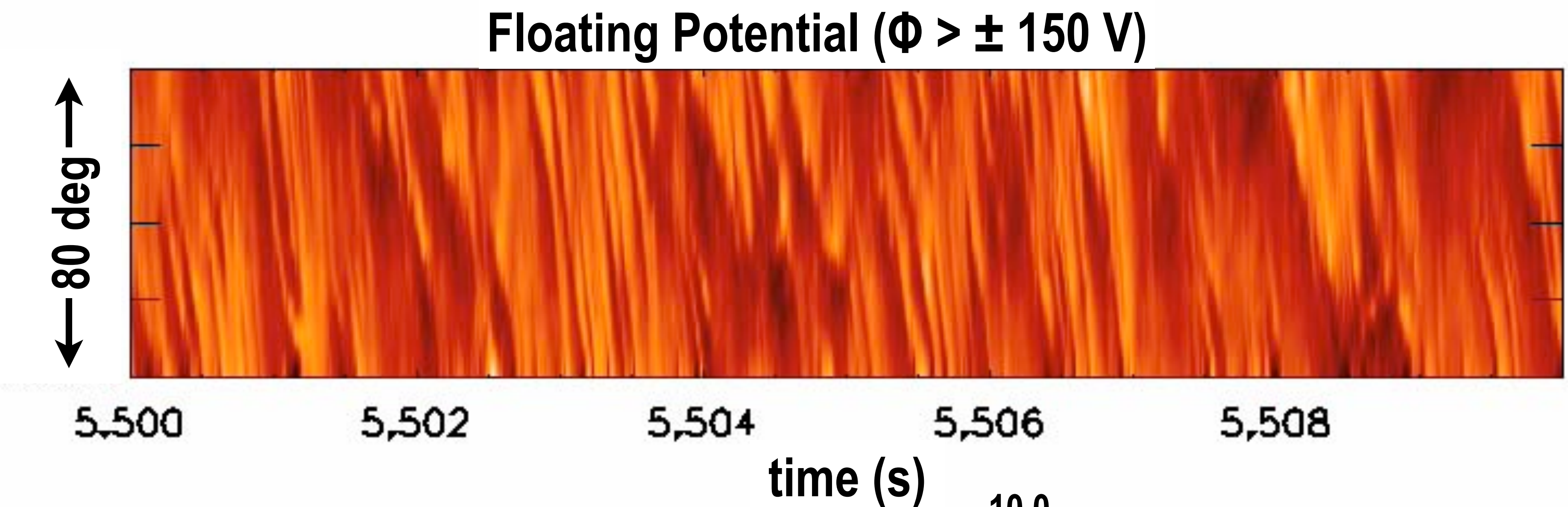


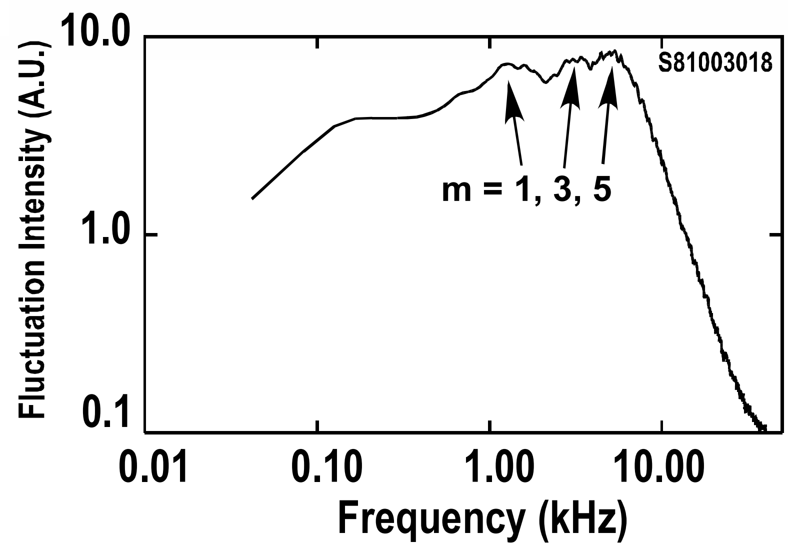
- *Good diagnostic coverage for low-frequency, long-wavelength fluctuations*

Floating Potential Probe Array

- Edge floating potential oscillations
- 4 deg spacing @ 1 m radius
- 24 probes
- Very long data records for excellent statistics!!

Floating Potential Probe Array





15 kW High-β Discharge

$\omega \sim \Omega\, m = \Omega R\, k$, with

$\Omega/2\pi \sim 1$ kHz

Low-Frequency Fluctuations are Observed throughout Plasma and Probably Cause Naturally Peaked Profiles

- **Low-frequency fluctuations (*f* ~ 1 kHz and < 20 kHz) are observed with edge probes, multiple photodiode arrays, µwave interferometry, and fast video cameras.**
- **The structure of these fluctuations are complex, turbulent, and still not well understood.**
- **Edge fluctuations can be intense (*E* ~ 200 V/m) and are dominated by long-wavelength modes that rotate with the plasma at 1-2 kHz**
- **High-speed digital records many seconds long enable analysis of turbulent spectra in a single shot. We find the edge fluctuations are characteristic of viscously-damped 2D interchange turbulence.**

Turbulent Radial Diffusion Implies an Inward Pinch

- Turbulent particle pinch links magnetic geometry and particle transport
- When flux-tube volume, $\delta V(\psi)$, varies rapidly with radius, then the turbulent pinch is large

$$\frac{\partial N}{\partial t} = \langle S \rangle + \frac{\partial}{\partial \psi} D \frac{\partial N}{\partial \psi}$$

Look!

$$\Gamma_\psi = -D \frac{\partial N}{\partial \psi} = -D \delta V \frac{\partial \langle n \rangle}{\partial \psi} + \boxed{V_\psi \langle n \rangle}$$

$$\text{where } V_\psi \equiv -D \frac{\partial \delta V}{\partial \psi}$$

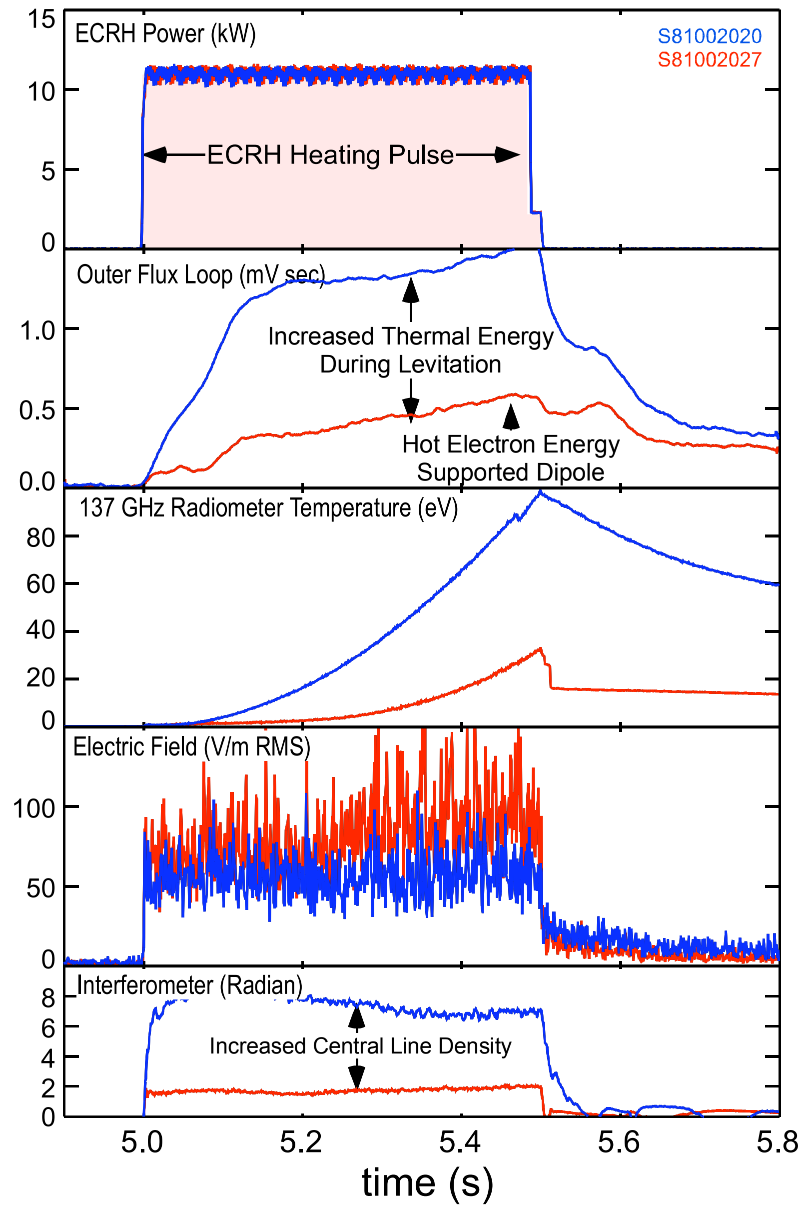
This is Big

LDX:
$D \approx 0.047$ Weber^2/s
V (pinch) ~ 45 m/s (core) and 400 m/s (edge)

Transport Studies Requires Measurements of both Sources and Fluxes

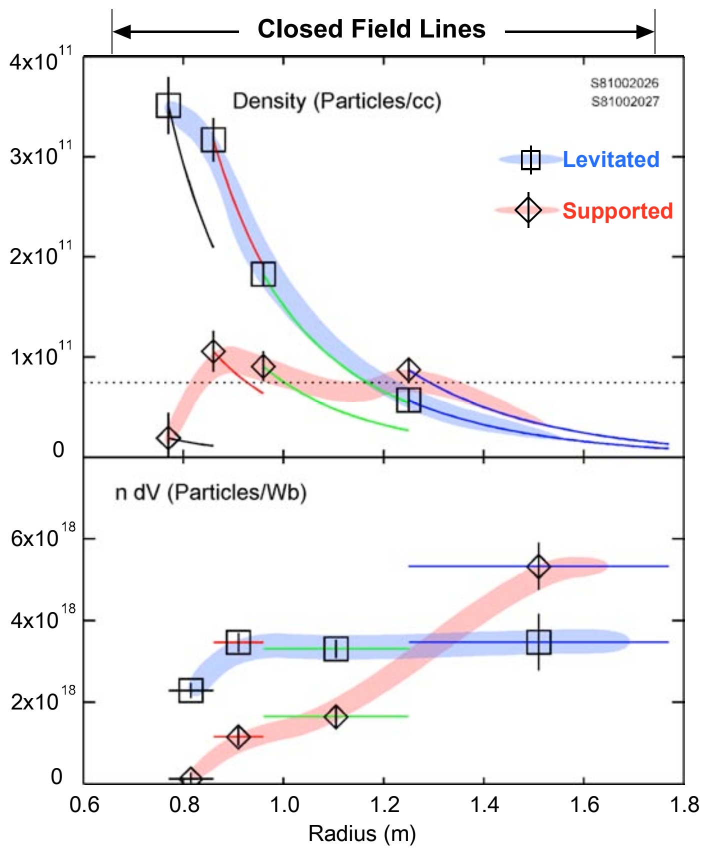
- Levitation vs. Supported comparisons provide an opportunity to directly observe the effects of turbulent transport, as the parallel losses are switched off/on.
- Short 1/2 second heating pulses minimize influence of hot electrons on plasma dynamics.
- Turbulent fluctuations are established quickly as the ECRH is switched on. Fluctuations diminish after ECRH is switched off.

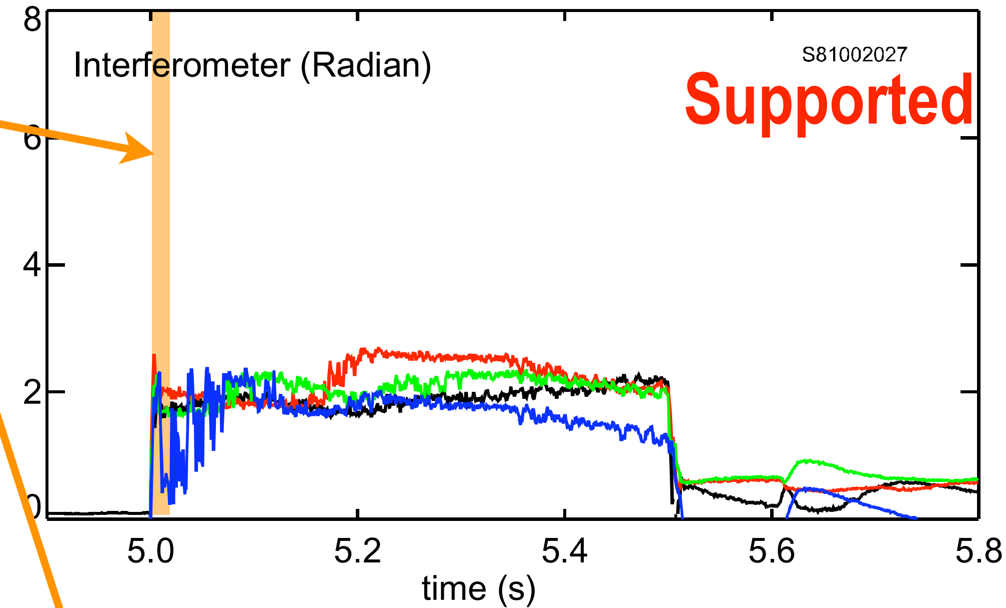
(a) Short Half-Second Heating Pulse

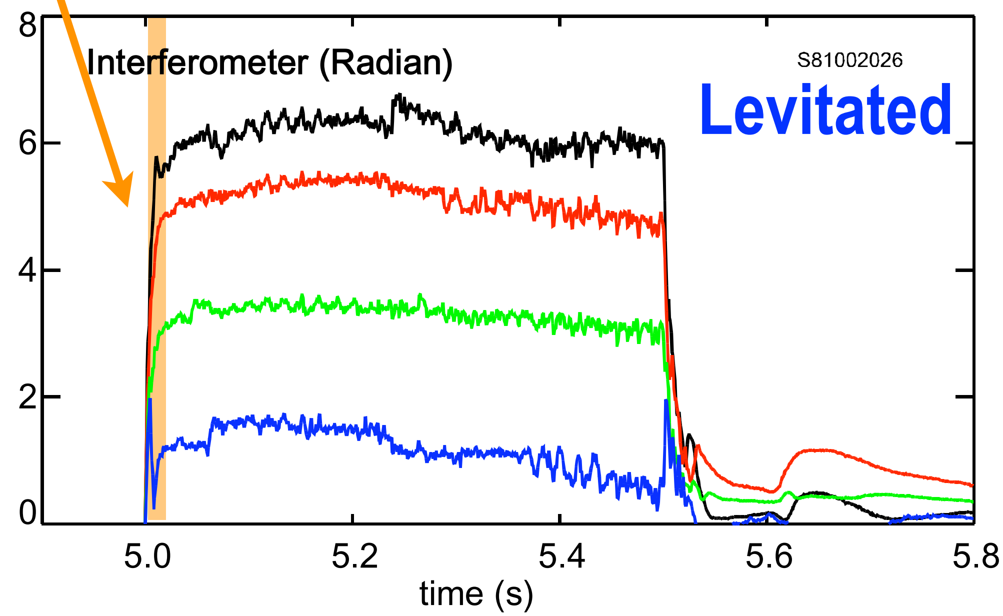


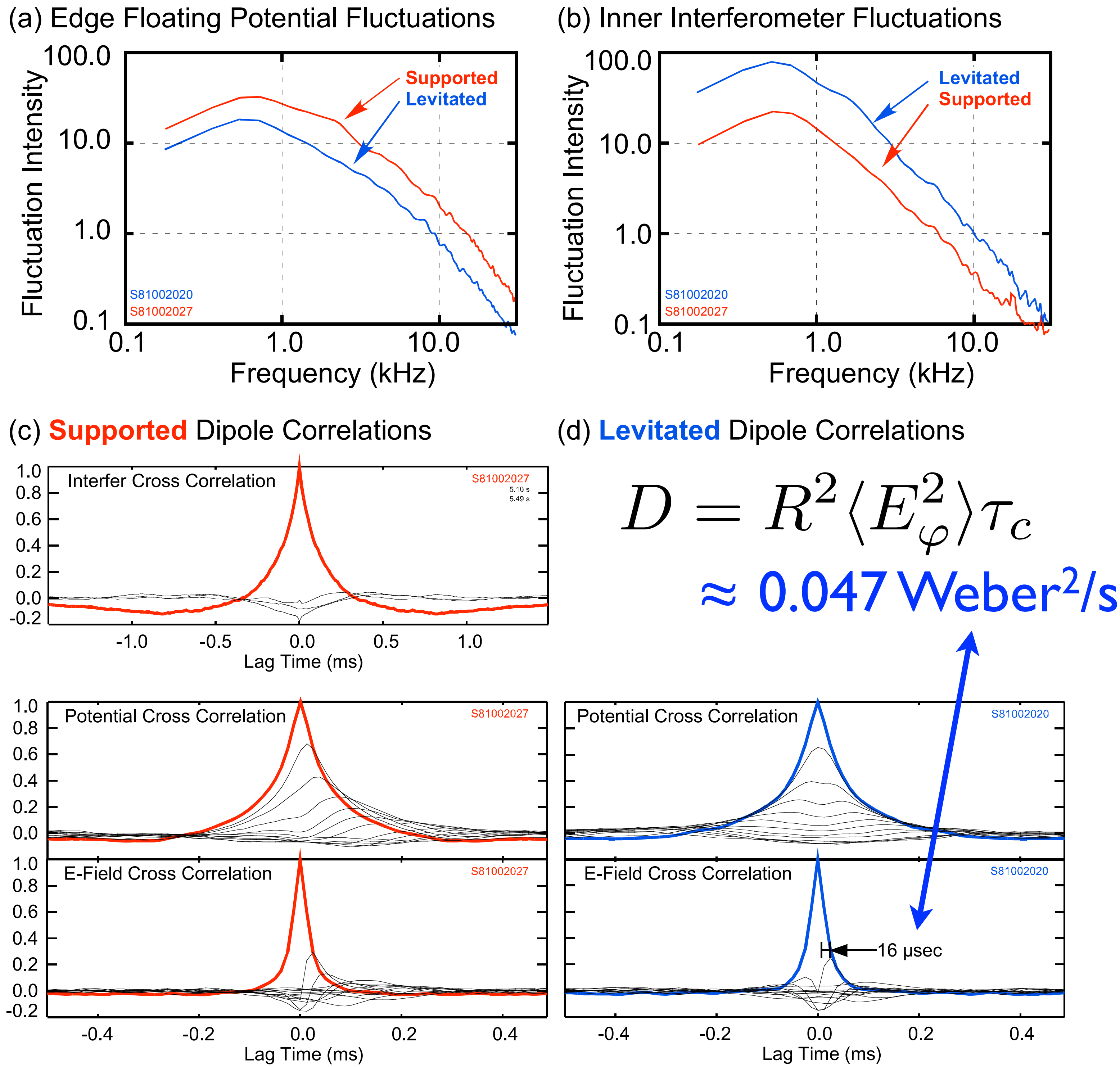
Naturally Peaked Profiles Established Rapidly

Short 20 msec Density Rise









$$D = R^2 \langle E_\varphi^2 \rangle \tau_c$$

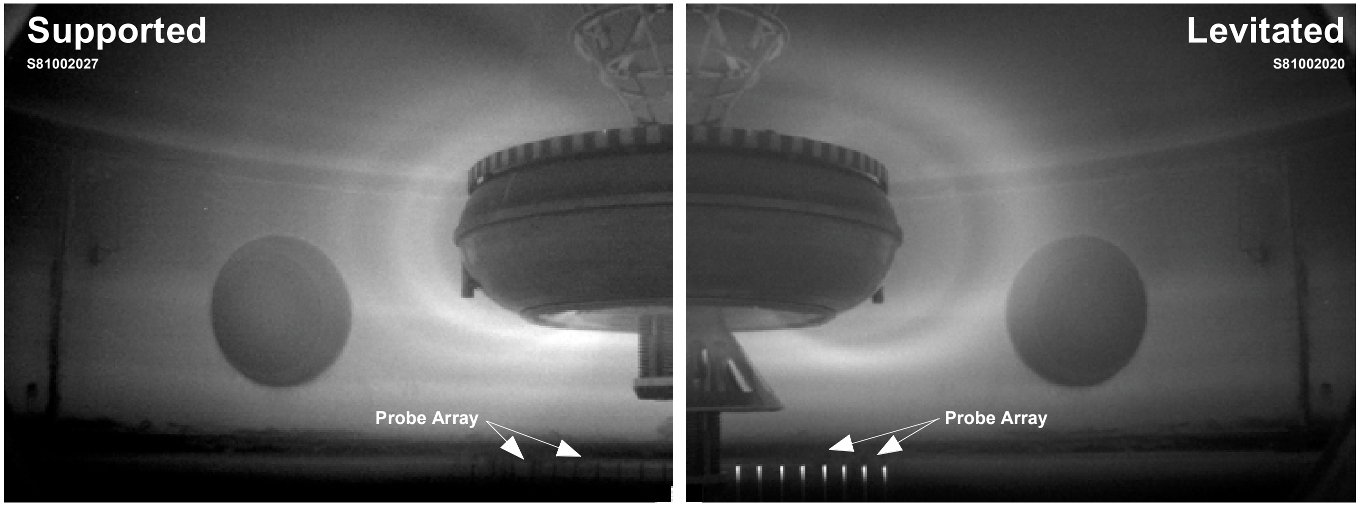
$\approx$ 0.047 Weber^2/s

Source is approx uniform during supported coil...

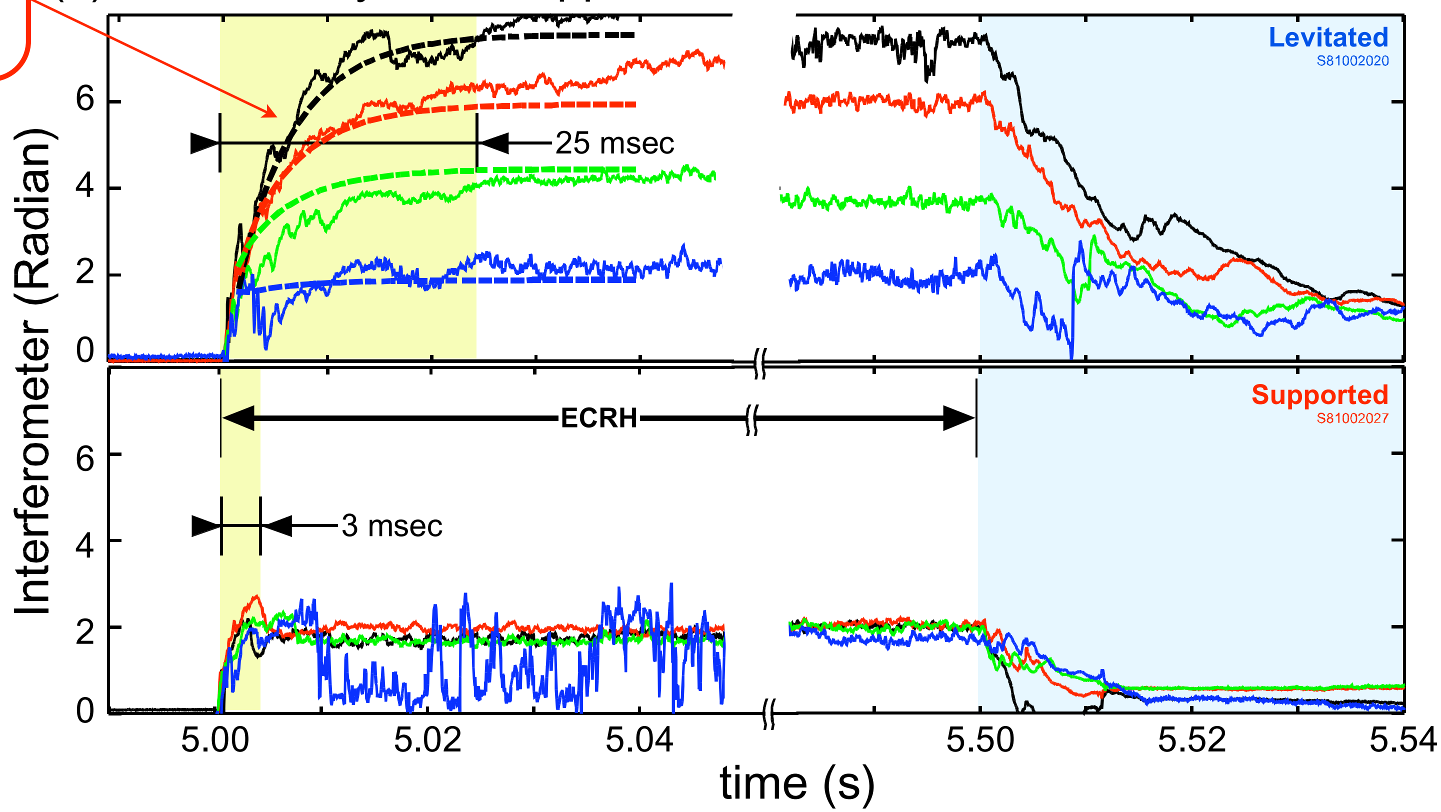
$$\boxed{\frac{\partial N}{\partial t} = \frac{\partial}{\partial \psi} D \frac{\partial N}{\partial \psi}}$$

with D uniform and measured at edge

(b) Visible Light from Supported and Levitated Plasma



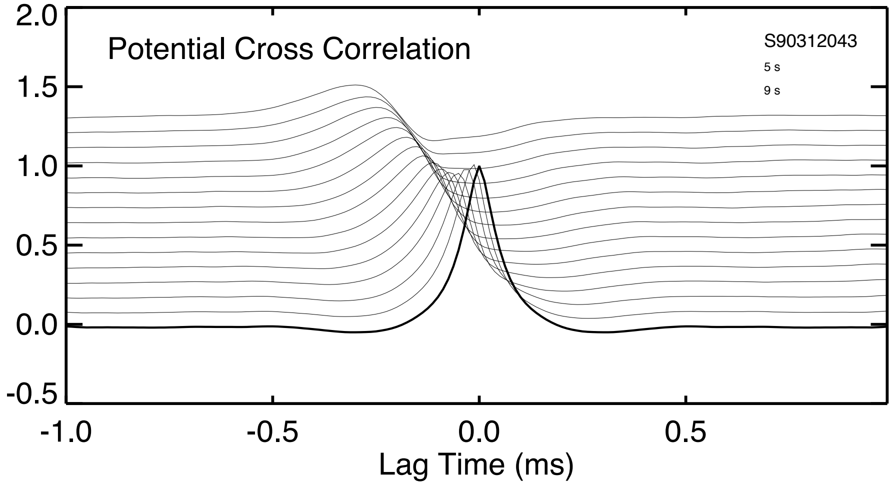
(c) Line Density from Supported and Levitated Plasma



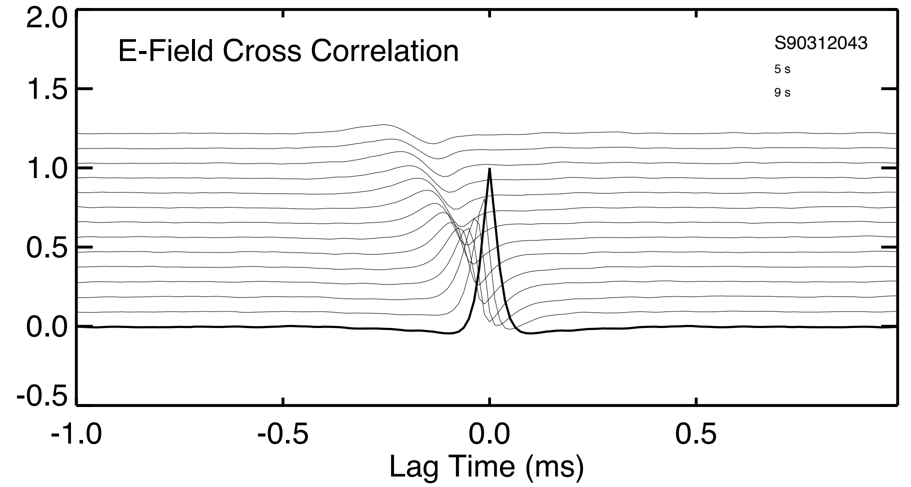
Decreased Fueling Pressure Allows Plasma Rotation and *increases* $D_{\psi\psi}$ *(edge)*

- *Examples measured with quasi-steady turbulence at 15 kW ECRH*
- 90312028 “low pressure” D_2 fueling: 1.0 μTorr, $\Omega/2\pi \sim 1.2$ kHz, $D_{\psi\psi} \sim 0.45\ (V \cdot s)^2/s$, $RE_\phi \sim 100\ V_{RMS}$, and $\tau_{cor} \sim 38$ μsec
- 90312025 “med/low pressure” D_2 fueling: 1.2 μTorr, $\Omega/2\pi \sim 0.9$ kHz, $D_{\psi\psi} \sim 0.80\ (V \cdot s)^2/s$, $RE_\phi \sim 110\ V_{RMS}$, and $\tau_{cor} \sim 67$ μsec
- 90312043 “medium pressure” D_2 fueling: 1.3 μTorr, $\Omega/2\pi \sim 0.56$ kHz, $D_{\psi\psi} \sim 0.39\ (V \cdot s)^2/s$, $RE_\phi \sim 85\ V_{RMS}$, and $\tau_{cor} \sim 54$ μsec
- 90312022 “medium pressure” He fueling: 3.8 μTorr, $\Omega/2\pi \sim 0.15$ kHz, $D_{\psi\psi} \sim 0.76\ (V \cdot s)^2/s$, $RE_\phi \sim 70\ V_{RMS}$, and $\tau_{cor} \sim 160$ μsec

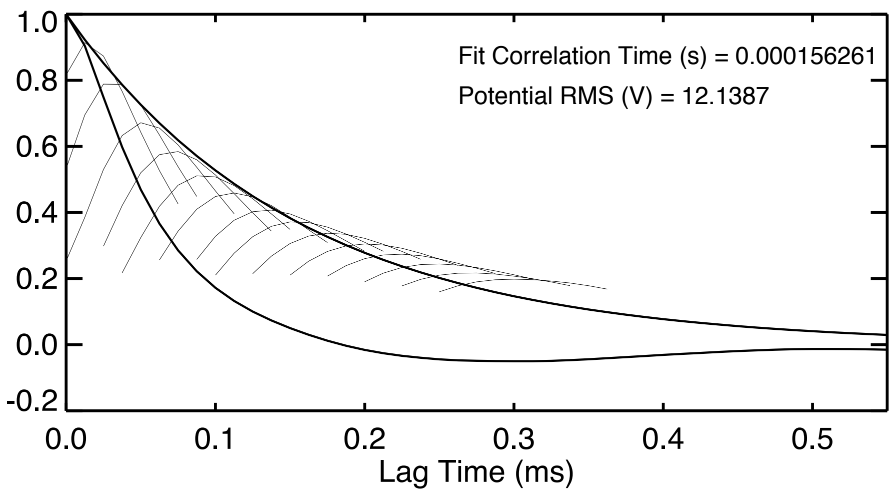
90312043



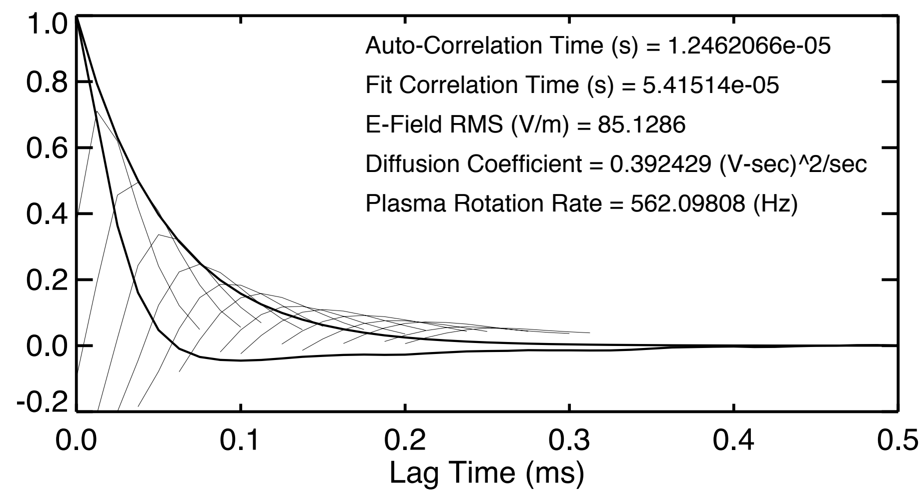
Potential Cross Correlation
S90312043
5 s
9 s
Lag Time (ms)



E-Field Cross Correlation
S90312043
5 s
9 s
Lag Time (ms)

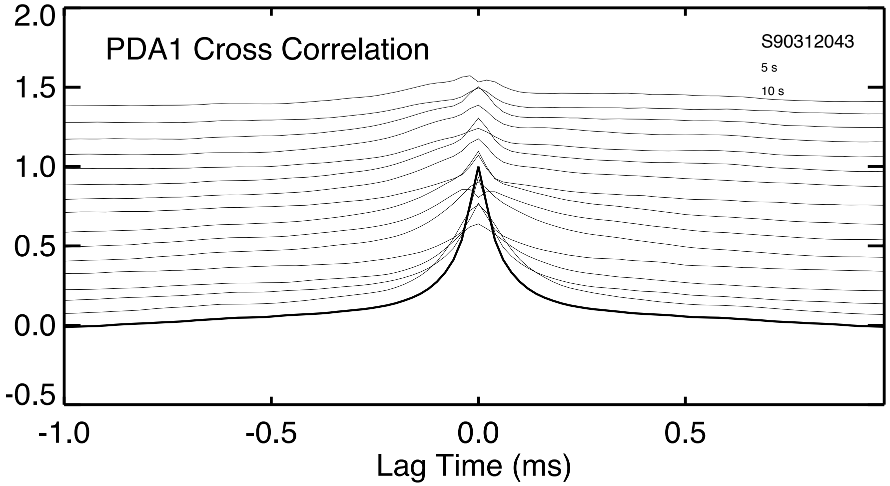


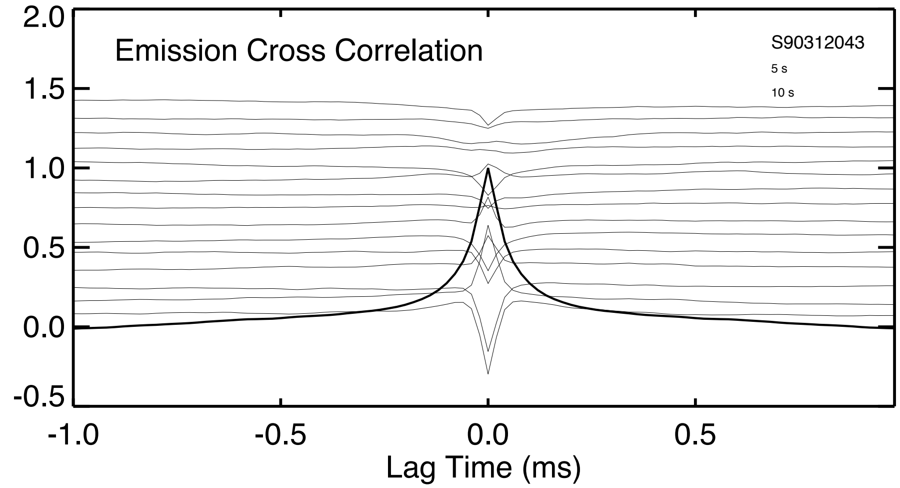
Fit Correlation Time (s) = 0.000156261
Potential RMS (V) = 12.1387
Lag Time (ms)

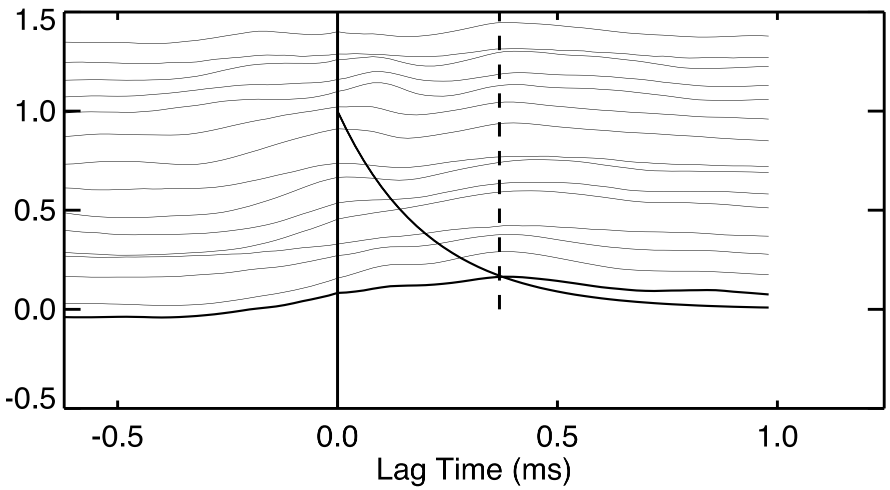


Auto-Correlation Time (s) = 1.2462066e-05
Fit Correlation Time (s) = 5.41514e-05
E-Field RMS (V/m) = 85.1286
Diffusion Coefficient = 0.392429 (V-sec)^2/sec
Plasma Rotation Rate = 562.09808 (Hz)
Lag Time (ms)

90312043







Time MaxCorrelation = 0.380 0.380 0.380 0.400 0.440 0.480 0.440 0.480 (ms)

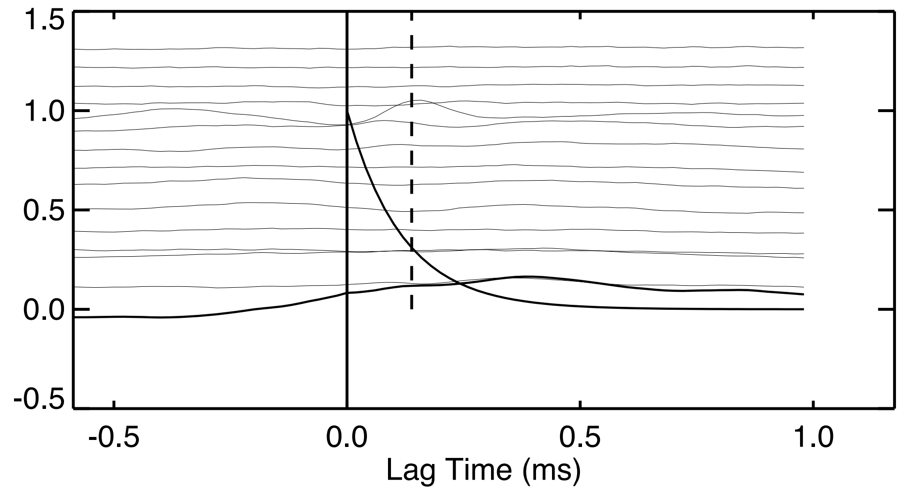
Time MaxCorrelation = 0.400 0.380 0.080 0.100 0.400 0.400 0.380 (ms)

Fit-Correlation Time (ms) = 0.211 0.235 0.229 0.205 0.293 0.273 0.267 0.223

Fit-Correlation Time (ms) = 0.241 0.238 0.051 0.056 0.231 0.171 0.189

Plasma Rotation Rate = 658. 658. 658. 625. 568. 521. 568. 521. (Hz)

Plasma Rotation Rate = 625. 658. 3125. 2500. 625. 625. 658. (Hz)



Time MaxCorrelation = 0.380 0.360 0.420-1.000-0.160-0.200-0.220 0.360 (ms)

Time MaxCorrelation = 0.460 0.080 0.160 0.260 0.400 0.560 0.220 (ms)

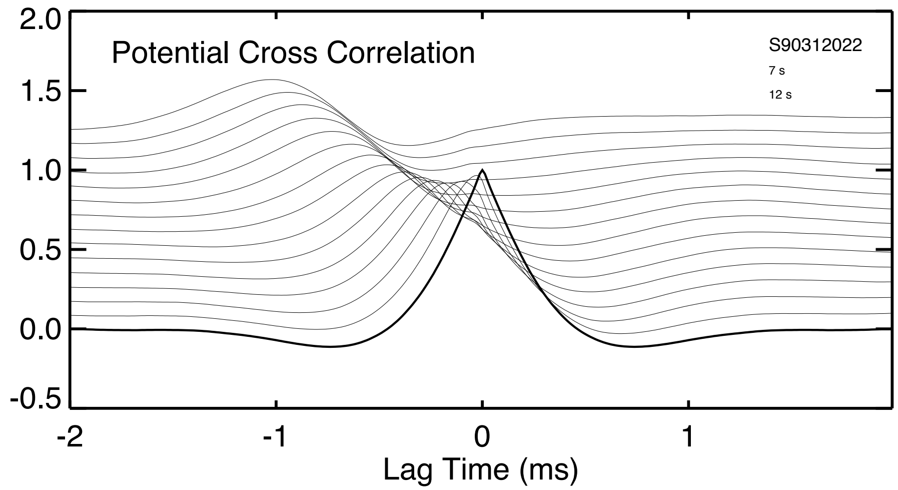
Fit-Correlation Time (ms) = 0.211 0.131 0.198 0.267 0.047 0.075 0.096 0.137

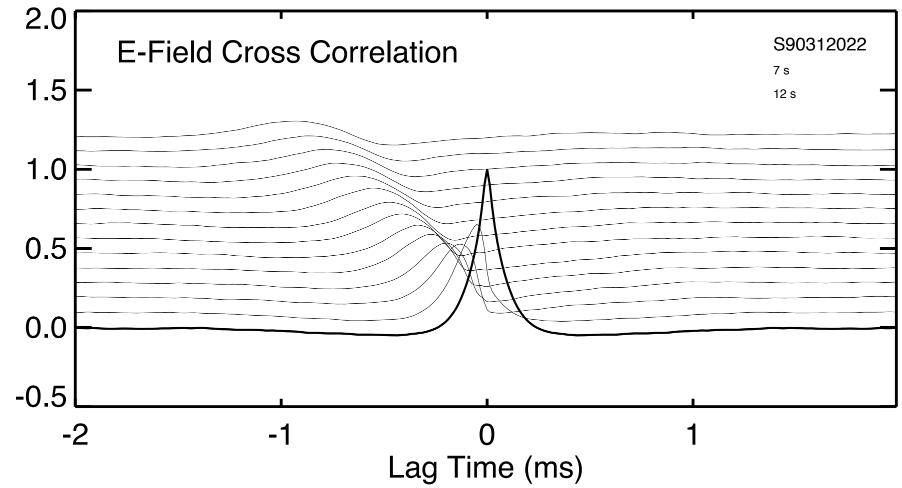
Fit-Correlation Time (ms) = 0.194 0.036 0.074 0.064 0.083 0.114 0.050

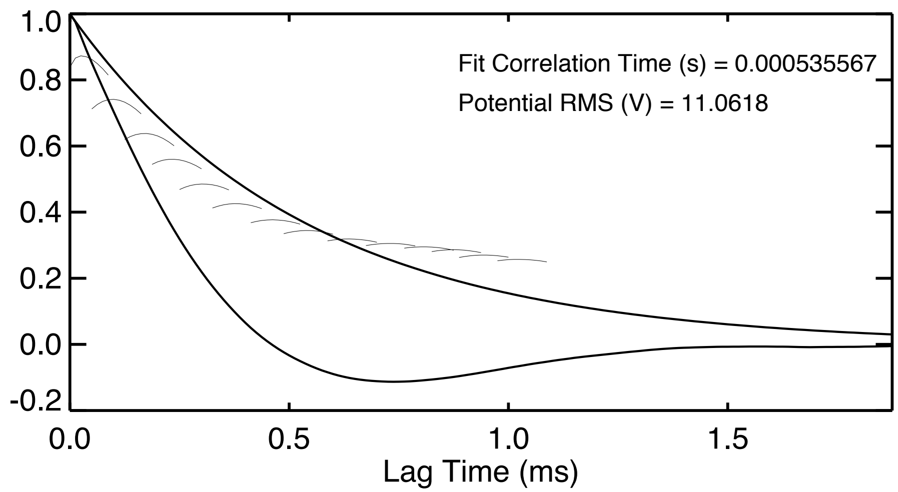
Plasma Rotation Rate = 658. 694. 595. -250.-1562.-1250.-1136. 694. (Hz)

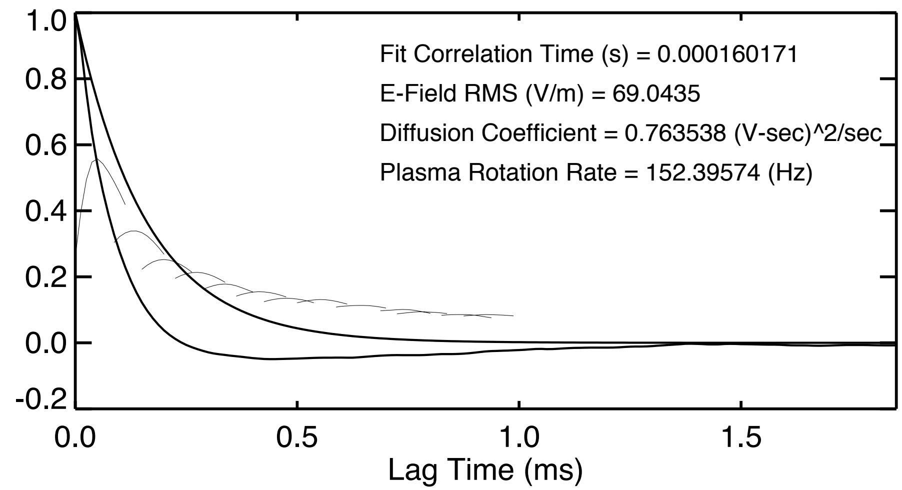
Plasma Rotation Rate = 543. 3125. 1562. 962. 625. 446. 1136. (Hz)

90312022 (He)

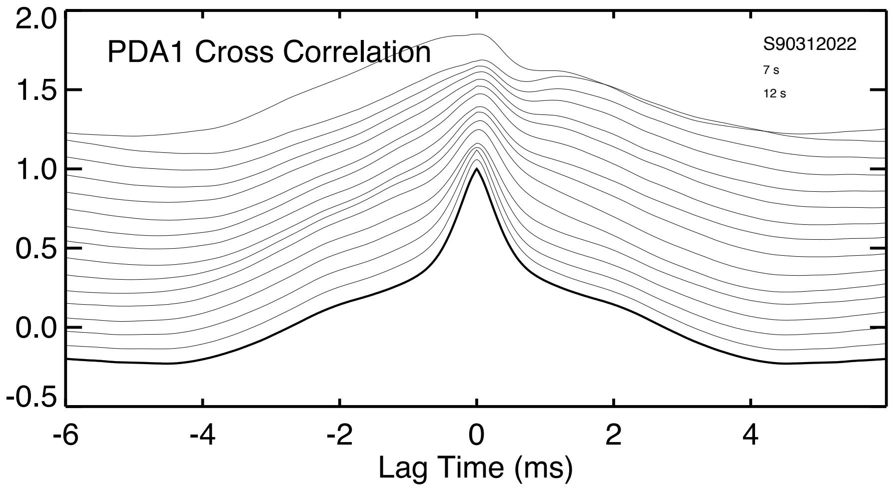

Potential Cross Correlation
S90312022
7 s
12 s
2.0
1.5
1.0
0.5
0.0
-0.5
-2
-1
0
1
Lag Time (ms)

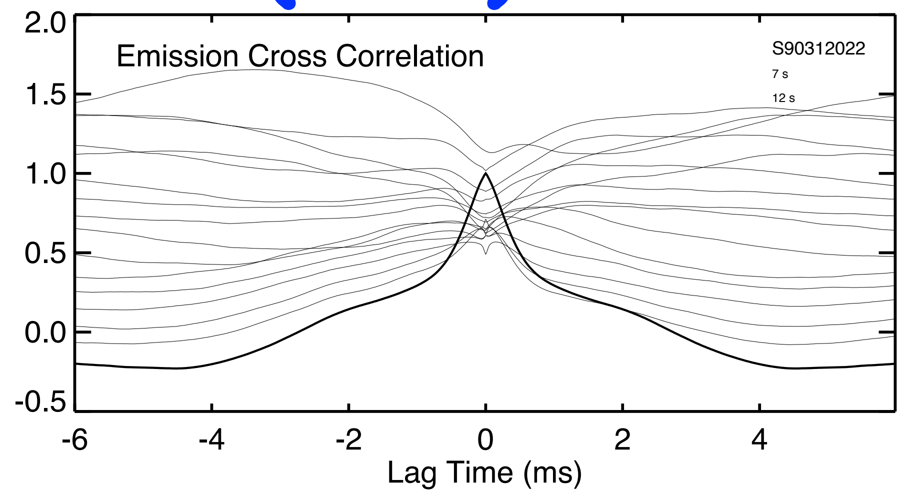

E-Field Cross Correlation
S90312022
7 s
12 s
2.0
1.5
1.0
0.5
0.0
-0.5
-2
-1
0
1
Lag Time (ms)

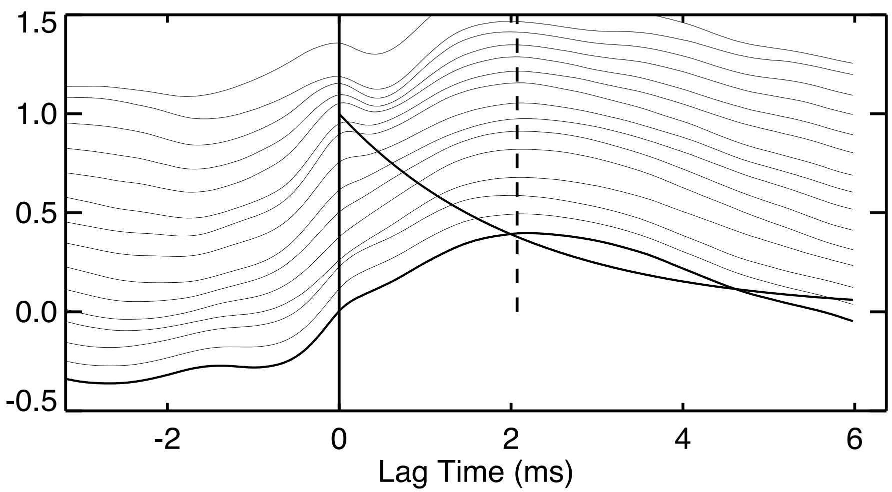

Fit Correlation Time (s) = 0.000535567
Potential RMS (V) = 11.0618
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0.0
0.5
1.0
1.5
Lag Time (ms)

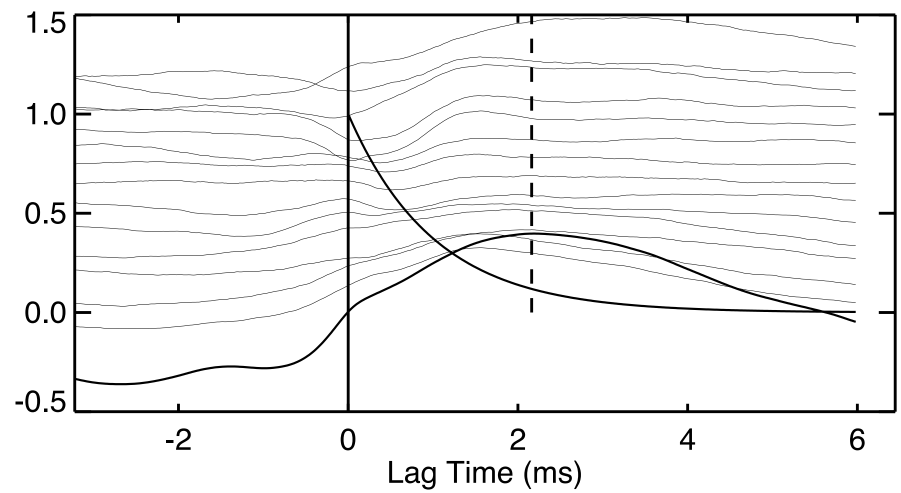

Fit Correlation Time (s) = 0.000160171
E-Field RMS (V/m) = 69.0435
Diffusion Coefficient = 0.763538 (V-sec)^2/sec
Plasma Rotation Rate = 152.39574 (Hz)
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0.0
0.5
1.0
1.5
Lag Time (ms)

90312022 (He)









Time MaxCorrelation = 2.200 2.120 2.000 2.100 2.140 2.120 2.180 2.120 (ms)
Time MaxCorrelation = 2.100 2.100 2.060 2.020 1.980 1.960 1.860 (ms)
Fit-Correlation Time (ms) = 2.385 2.319 2.184 2.276 2.648 2.604 2.465 2.303
Fit-Correlation Time (ms) = 2.332 2.120 1.962 1.758 1.593 1.408 1.635
Plasma Rotation Rate = 114. 118. 125. 119. 117. 118. 115. 118. (Hz)
Plasma Rotation Rate = 119. 119. 121. 124. 126. 128. 134. (Hz)

Time MaxCorrelation = 2.200 1.560 1.540 2.160 1.960 1.800 3.820 2.160 (ms)
Time MaxCorrelation = 1.480 3.860 1.560 1.640 1.640 1.560 3.480 (ms)
Fit-Correlation Time (ms) = 2.385 1.070 0.983 1.082 1.010 0.709 1.132 0.641
Fit-Correlation Time (ms) = 0.489 1.156 0.613 0.589 0.784 0.586 1.992
Plasma Rotation Rate = 114. 160. 162. 116. 128. 139. 65. 116. (Hz)
Plasma Rotation Rate = 169. 65. 160. 152. 152. 160. 72. (Hz)

Turbulent Particle Pinch is associated with Turbulent Enstropy Pinch: Pressure Peaking

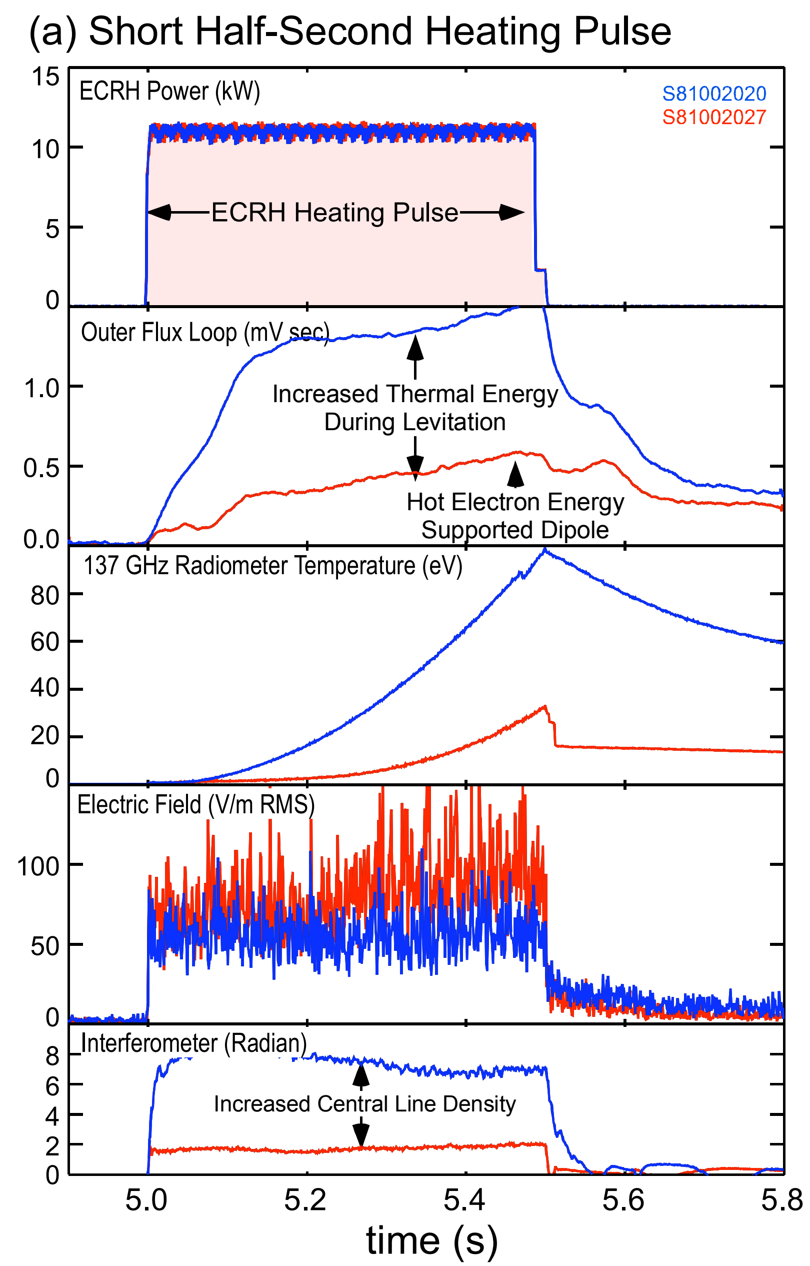
- Flux-tube density and enstropy density have identical dynamics for a plasma with an adiabatic closure, $G = P\delta V^{\gamma}$

$$
\begin{aligned}
\frac{\partial N}{\partial t} - \frac{\partial}{\partial \varphi}\left(N\frac{\partial \Phi}{\partial \psi}\right) + \frac{\partial}{\partial \psi}\left(N\frac{\partial \Phi}{\partial \varphi}\right) &= S \\
\frac{\partial G}{\partial t} - \frac{\partial}{\partial \varphi}\left(G\frac{\partial \Phi}{\partial \psi}\right) + \frac{\partial}{\partial \psi}\left(G\frac{\partial \Phi}{\partial \varphi}\right) &= H
\end{aligned}
$$

- (N, G) ~ constant implies peaked density and pressure profiles

- Edge T_e ~ 15 eV, implies central T_e ~ 500 eV with measured diamagnetism and measured density profile

- Thermal stored energy of 60 J (this example levitated discharge, 2 μTorr D_2)



Works in Progress…

- Improve diagnostics of density evolution and particle source profile
- Understand transport boundaries: inner and outer edges
- Diagnose density profile transients
- Additional comparisons between levitated and supported discharges
- Improve internal fluctuation structure measurements
- Measure and understand enstropy dynamics and evolution
- Study and understand transport rate changes as a function of plasma, fueling, and power variations.

Summary

- The mechanics of magnetic levitation is **proven reliable**.
- Levitation eliminates parallel particle losses and allows a **dramatic peaking of central density**.

  LDX has demonstrated the formation of natural density profiles in a laboratory dipole plasma and **the applicability of space physics to fusion science.**
- Fluctuations of density and potential show **large-scale circulation** that is the likely cause of measured inward pinch.
- Increased stored energy consistent with adiabatic profiles: ***a necessary physics requirement for dipole fusion.***