Reconstruction of Pressure Profile Evolution during Levitated Dipole Experiments

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Abstract

Magnetic levitation of the LDX superconducting dipole causes significant changes in the measured diamagnetic flux and what appears to be fascinating temporal evolution of plasma diamagnetic current. This poster describes the reconstruction of plasma current and plasma pressure profiles from external measurements of the equilibrium magnetic field, which vary substantially as a function of time. Previous free-boundary reconstructions of plasma equilibrium [1] showed the plasma to be anisotropic and highly peeked at the location of the cyclotron resonance of the microwave heating sources. Reconstructions of the peaked plasma pressures confined by a levitated dipole incorporate the small axial motion of the dipole (+/- 5 mm), time varying levitation coil currents, eddy currents flowing in the vacuum vessel, constant magnetic flux linking the superconductor, and new flux loops located near the hot plasma in order to closely couple to plasma current and dipole current variations.

[1] I. Karim, et al., "Equilibrium reconstruction of anisotropic pressure profile in the levitated dipole experiment." J. Fusion Energy, **26** (2007) 99.

Key Points

- During magnetic levitation, **vertical motion** of the superconducting dipole, **changes in the levitation control** coil current, and **induced eddy currents** couple to magnetic diagnostics.
- We "self-calibrate" the magnetics using levitation current ramps and pre-programmed "jogs" of dipole's vertical position.
- Using data from the calibration shots, the induced eddy currents are calculated (and digitally "removed") by inverting coupled linear ODEs. This allows...
- Use of previous magnetic reconstruction methods.

Magnetic Detectors

The magnetic field is axisymmetric. The dipole motion is axial. The ith magnetic detector, whether a flux coil or a magnetic field vector component, can be designated by a location (R_i, Z_i). The magnetic signal detected by the ith detector is given by the equation

$$\begin{aligned} S_i(t) - S_i(0) &= G_{i,L}\left[I_L(t) - I_L(0)\right] + \sum_w G_{i,w} I_w(t) \\ &+ \left[G_{i,D}(t) I_D(t) - G_{i,D}(0) I_D(0)\right] + \sum_p G_{i,p}(t) I_p(t) \qquad (1) \end{aligned}$$

In Eq. 1, the signal equals the sum of the response from all coupled equilibrium, control, and vessel eddy currents in proportion to a Green's function, or mutual inductance. I_L is the levitation control current, I_w are the eddy currents flowing in the vessel, I_D is the dipole current, and I_p are the plasma ring currents.

Reconstruction

The wall eddy currents can be expressed as an expansion of orthogonal, and axisymmetric, "modes" with decreasing current decay times.

Since the flux linked by the superconducting dipole is constant, $I_D(t)$ can be determined simultaneously with the solution to Eq. 1 with knowledge of the dipole's axial position. The constant flux constraint is

$$0 = L_D I_D(t) + \sum_p M_p I_p(t) + M_L(t) I_L(t) + \sum_w M_w(t) I_w(t)\,. \qquad (2)$$

Eqs. 1 and 2 represent a set of simultaneous linear equations for the unknown currents, (I_p, I_D, I_w). Using only 15 working flux loops, the number of unknown currents could reach 15. In practice, some of the flux loop measurements are not independeny, and the number of unknown currents must be much smaller and determined by practice. A good choice should be two or three "plasma" current rings and one or two "wall eddy current modes." With $p \in \{1, 2\}$ and $w \in \{1\}$, there are four current unknowns including I_D. The least squares most likely values of these four currents can be determined from the 15 flux loops using singular value decomposition (SVD).

Outline

(1) Previous magnetic reconstruction results from LDX and space

(2) "Self calibration" using levitation control and dipole position ramps

(3) How much current in the superconducting dipole? Measurement using weight

(4) Estimating the diamagnetic plasma current

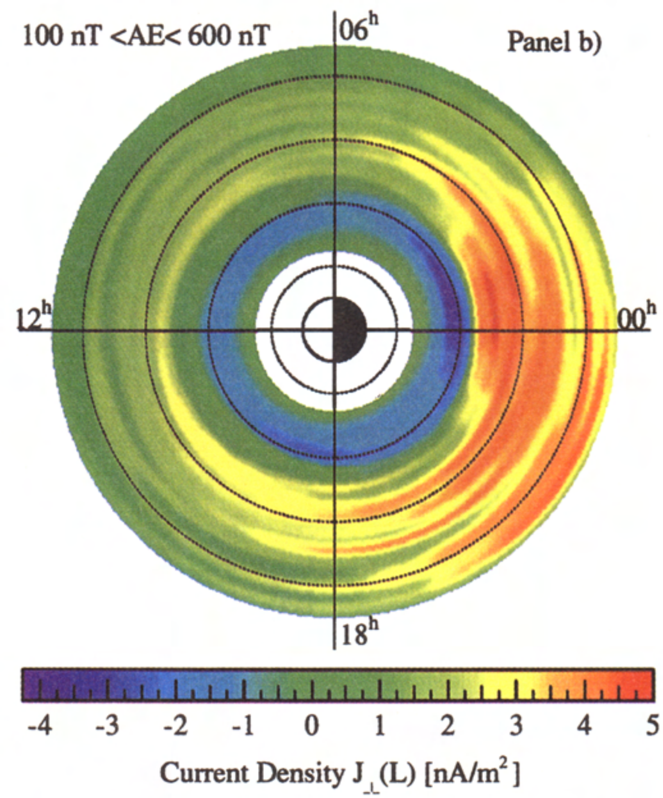
(5) Initial plasma pressure reconstruction. Best fit pressure is isotropic during levitation!

(1) Previous Results

- High beta plasmas created **like those found in magnetosphere**
- Anisotropic
- Required x-ray imaging to determine peak pressure
- Ring current ~ Plasma Stored Energy ($W_p \approx 170$ (J/kA) I_p)

Ring Current:
Trapped, High-β Protons (15-250 keV)

- Greatly intensified during geomagnetic storms
- $T_i \sim 7T_e$ and $P_\perp \sim 1.5\ P_{||}$
- Monthly storms: ~5 MA. (LDX: 3-4 kA) 10 MA storms few times a year.
- Current centered near $L \sim$ 4-5R_e; ΔL ~ 2.6R_e wide and Δz ~ 1.6R_e; *Not axisymmetric.*
- Curlometer during storms: $J_{RC} \sim 25$ nA/m^2 (Cluster II, 2005)



AMPTE/CCE-CHEM Measurements
Averaged over 2 years
(De Michelis, Daglis, Consolini, *JGR*, 1999)

D_{st} and the Dessler-Parker-Sckopke Relation

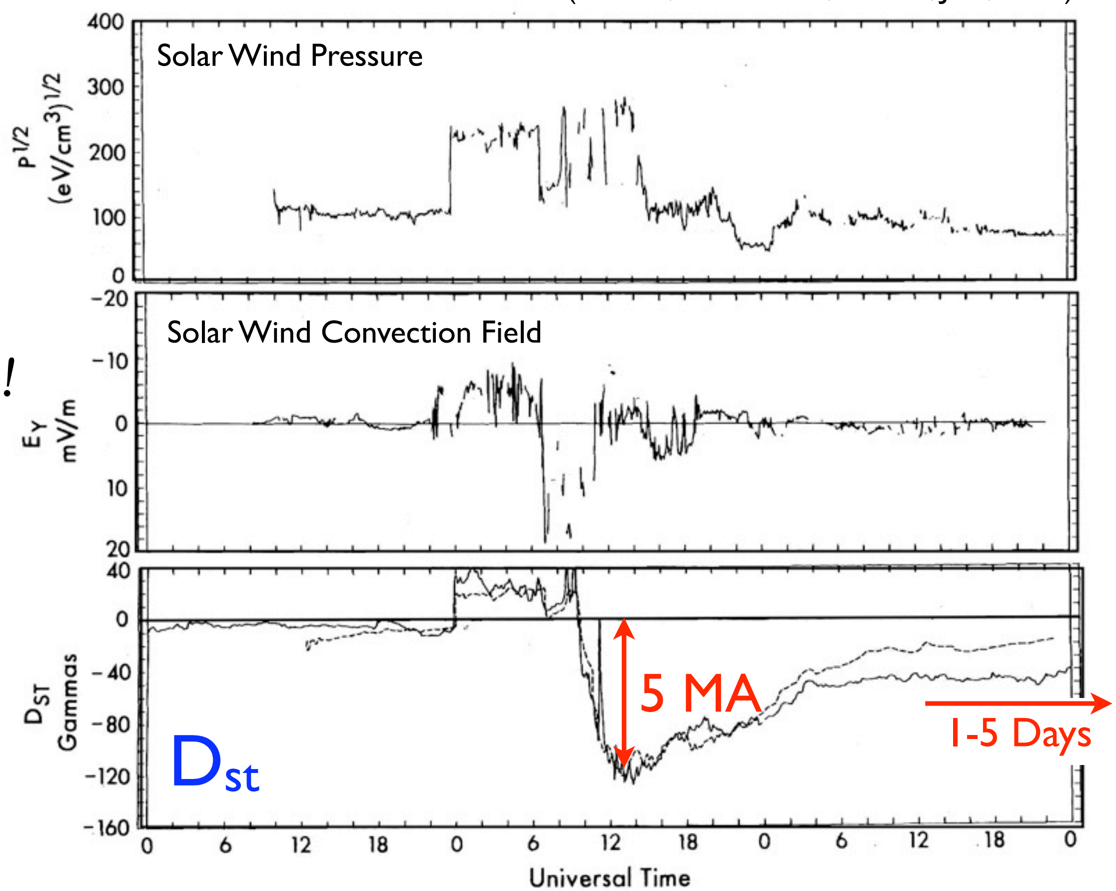
- Disturbed Storm Time Index (D_{st}):

  $\Delta B_H = (\mu_0/2) \times I_{RC}/R_{rc}$
  measured near equator
  plus Earth's induction fields!
  (LDX: $\Delta I_F \approx -0.25\ I_{rc}$)

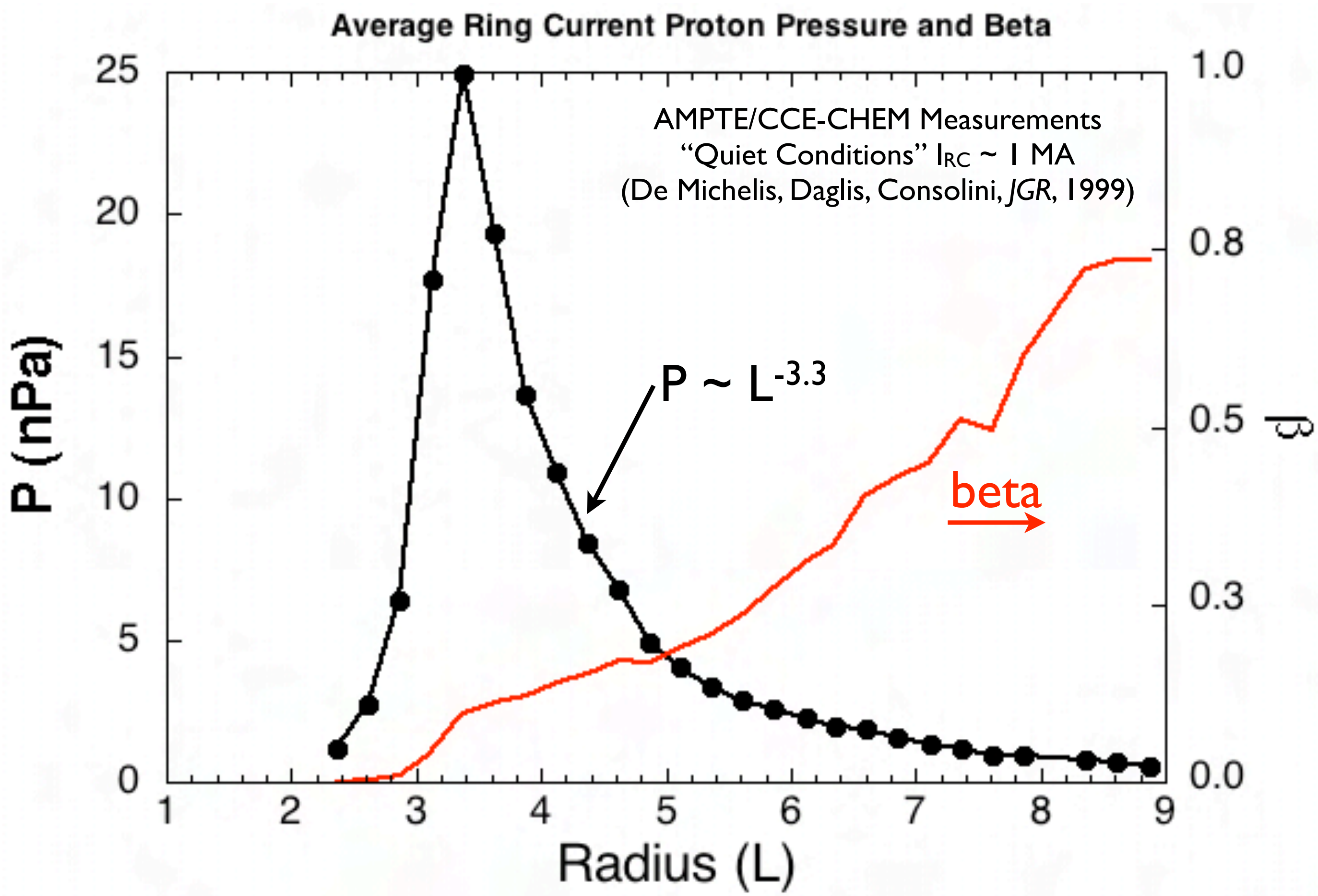
- Dessler-Parker-Sckopke:

  Energy = 0.54 GJ/A × I_{RC}
  (LDX: 0.12 J/A)

(Burton, McPherron, Russell, *JGR*, 1975)



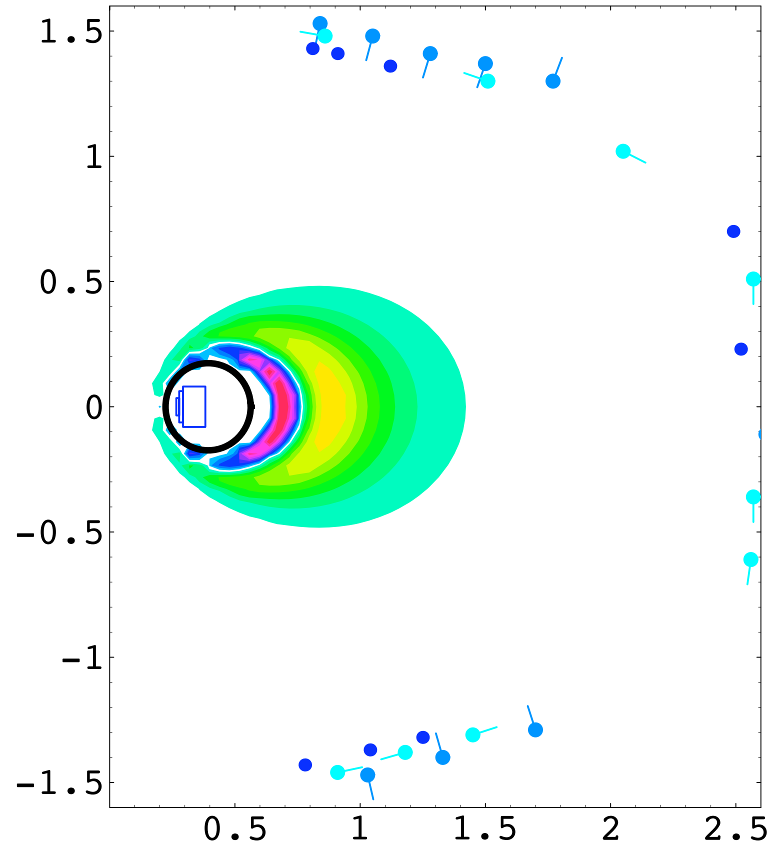
Centrally-Peaked Proton Pressure (Even with Plasma Sheet, Outer-Edge, Source!)



Where is the Ring Current?

(I. Karim, 2007)

$$\mathbf{J}_\perp = \frac{\mathbf{B} \times \nabla P_\perp}{B^2} + \frac{\mathbf{B} \times \kappa}{B^2}(P_{||} - P_\perp)$$



- 8 flux loops
- 9 normal-B sensors
- 9 tangential-B sensors
- Constant flux constraint on superconducting dipole
- Isotropic now ($P_\perp > P_{||}$ in future)
- 26 measurements;
  3 unknowns: (p_0, ψ_0, g) …

$$p(\psi) = p_0 \left(\frac{\psi - \psi_{fcoil}}{\psi_0 - \psi_{fcoil}} \right)^\alpha \left(\frac{\psi}{\psi_0} \right)^{4g}$$

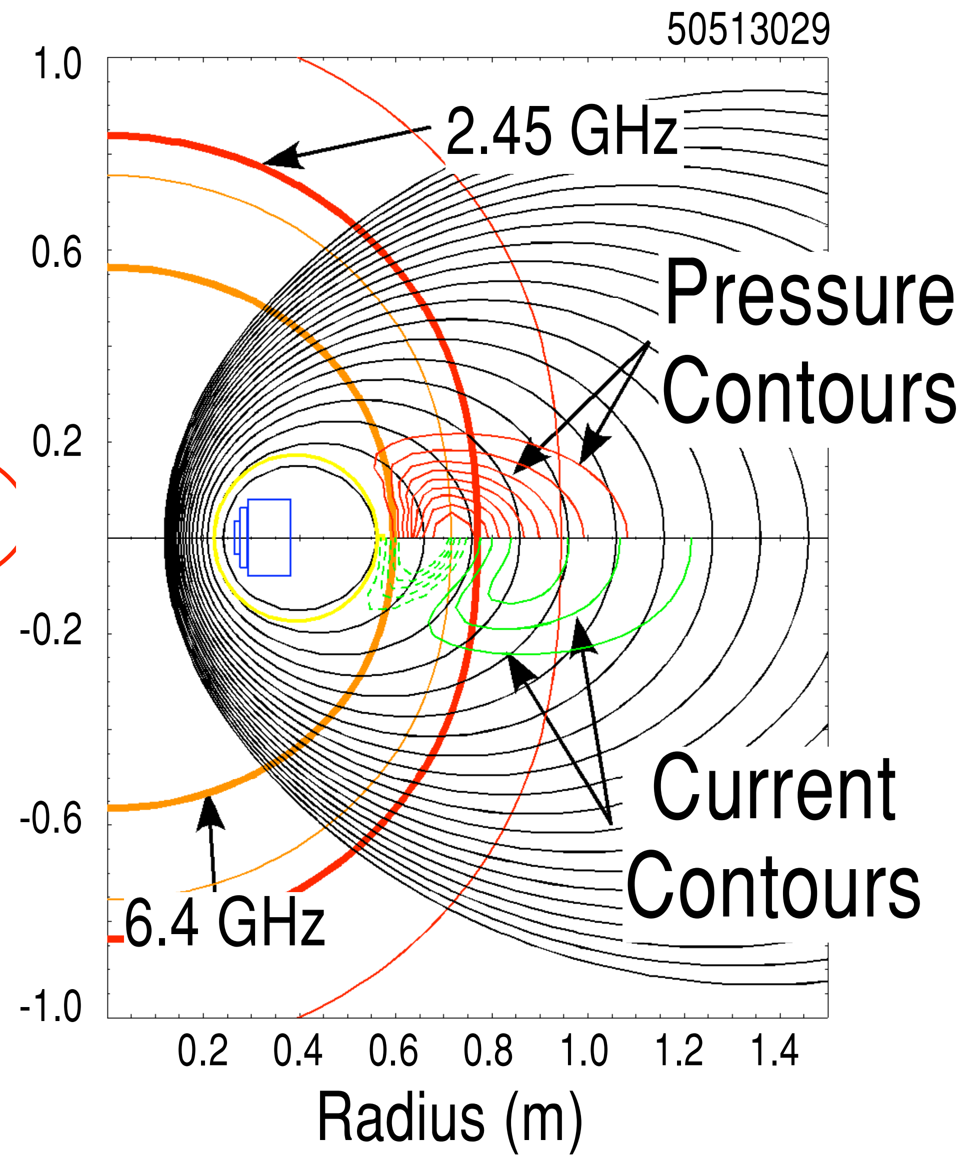
"Best Fit" Anisotropic Equilibrium: **Supported Dipole**

Parameter	Fit Value
χ^2	14.5942
Ip	3356.57
δIf	-738.437
p	2
P(perp)/P(||)	5
R(peak)	0.716667
γ	2.40741
γ/(5/3)	1.44444
Press(Rpeak)	594.78
J Centroid	1.23389
Moment (A m^2)	5251.09
Max Perp β	0.267272
Perp β(Rpeak)	0.115559
Avg Perp β	0.0383653
Plasma Volume	28.7984
Energy (J)	306.234
E/Ip (J/kA)	91.2342

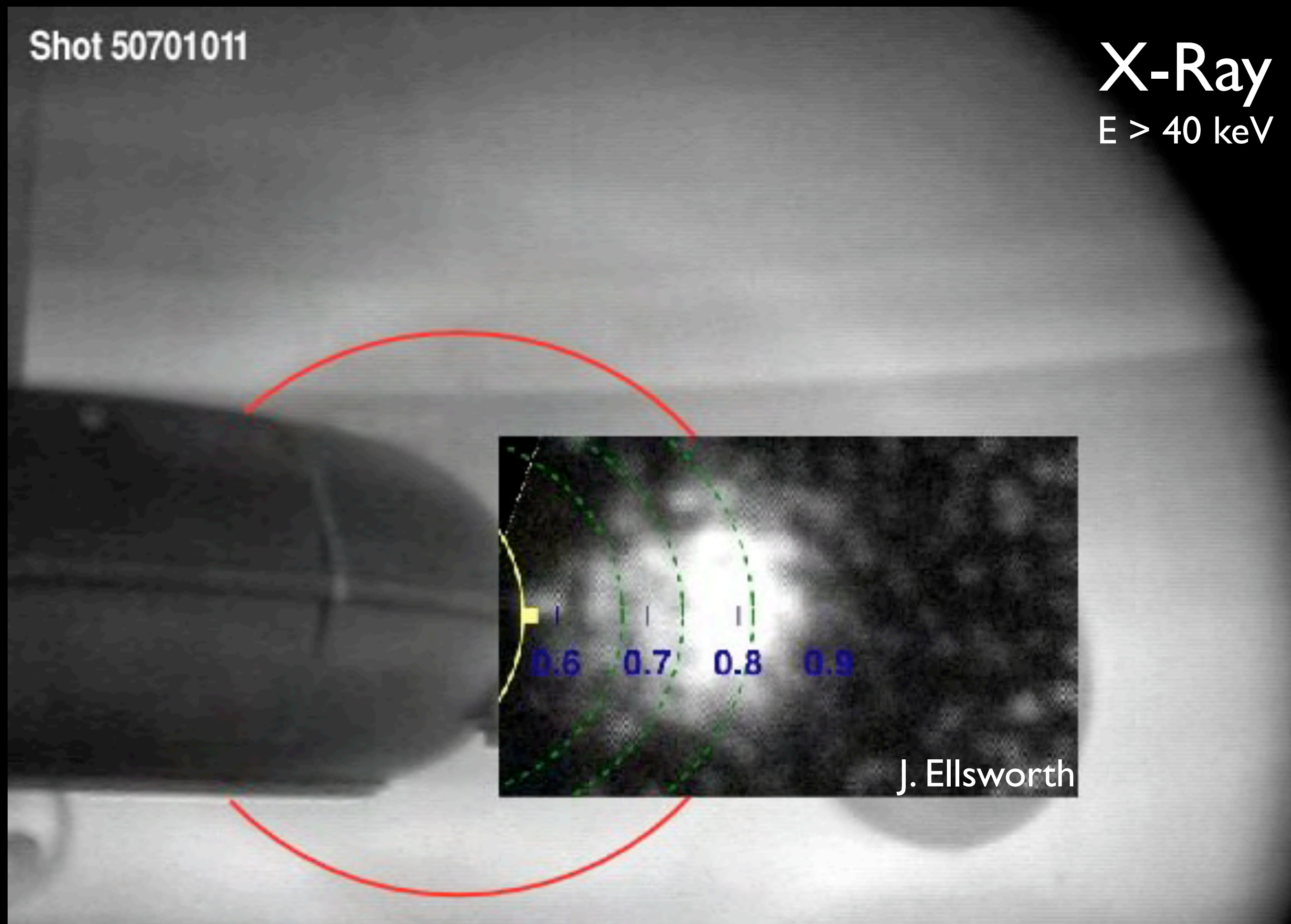
Steep Gradient!

High β!

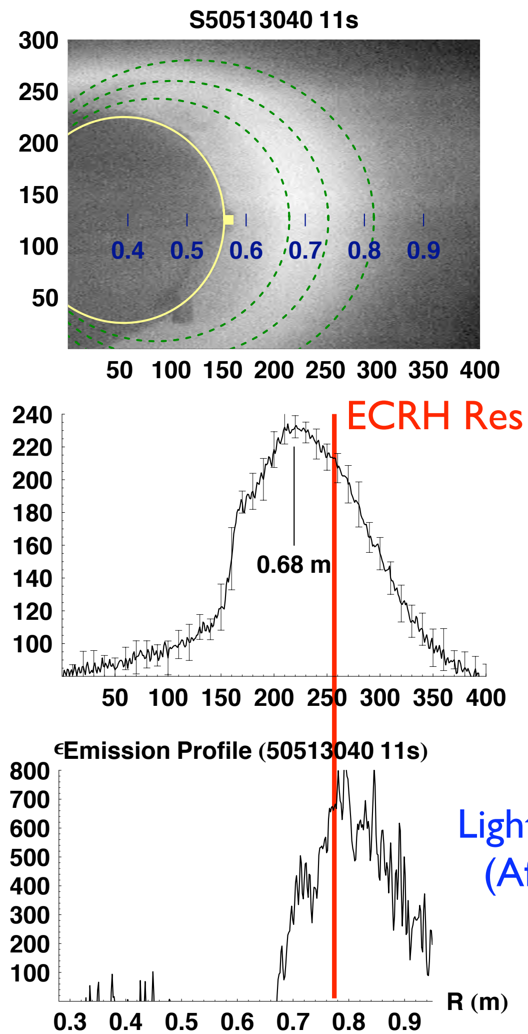
Peak pressure "in-between" 2.45 and 6.4 resonance.



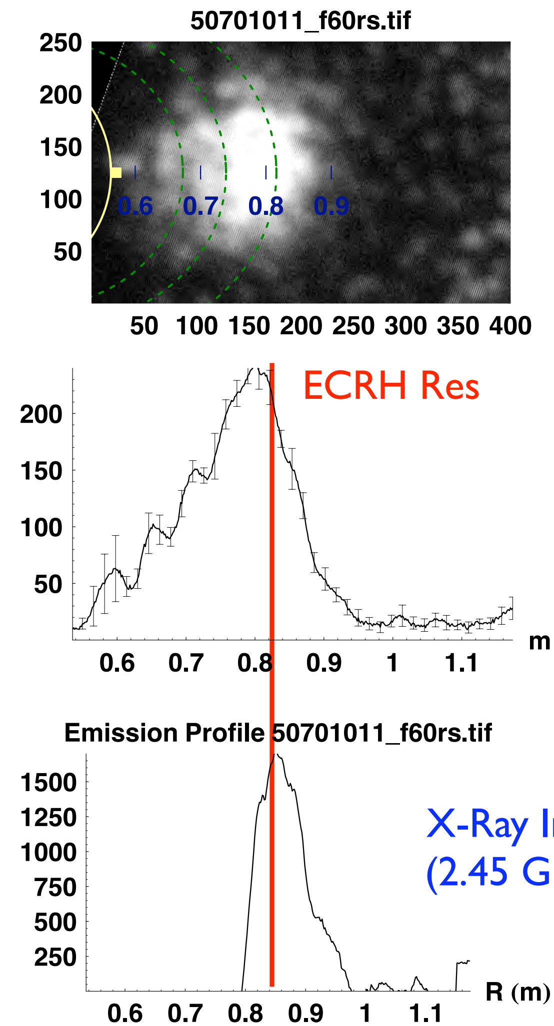
Where is the High-β Plasma?



Abel Inversion (Equatorial) Show Profiles Highly Peaked Near 2.45 GHz Resonance



Light Inversion (Afterglow)



X-Ray Inversion (2.45 GHz only)

High β Anisotropic Pressure Produced when Dipole is Mechanically Supported

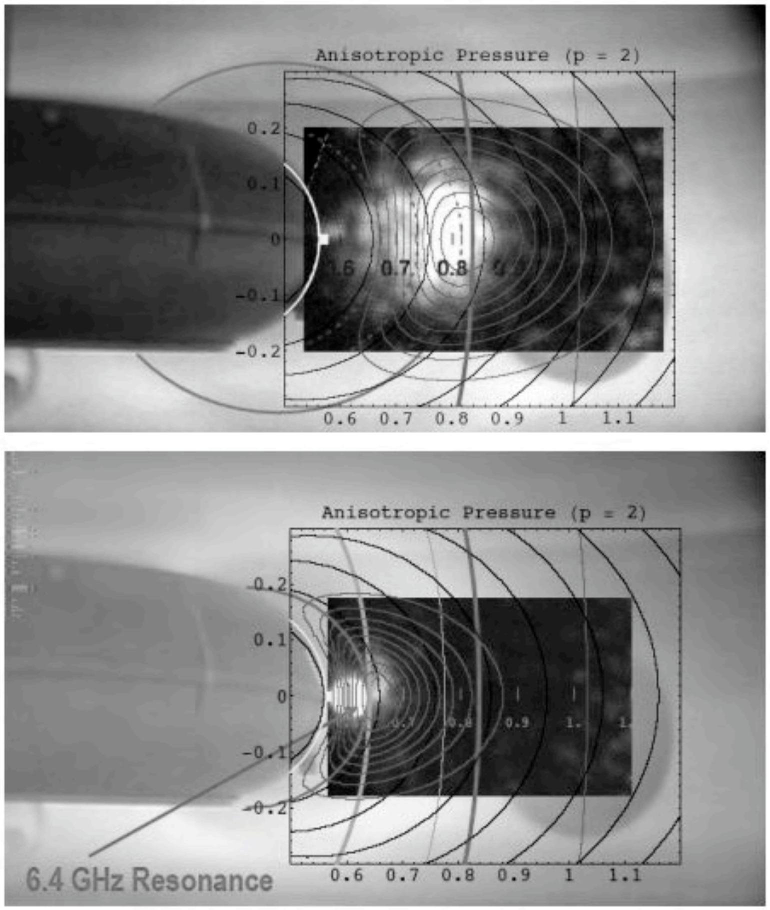
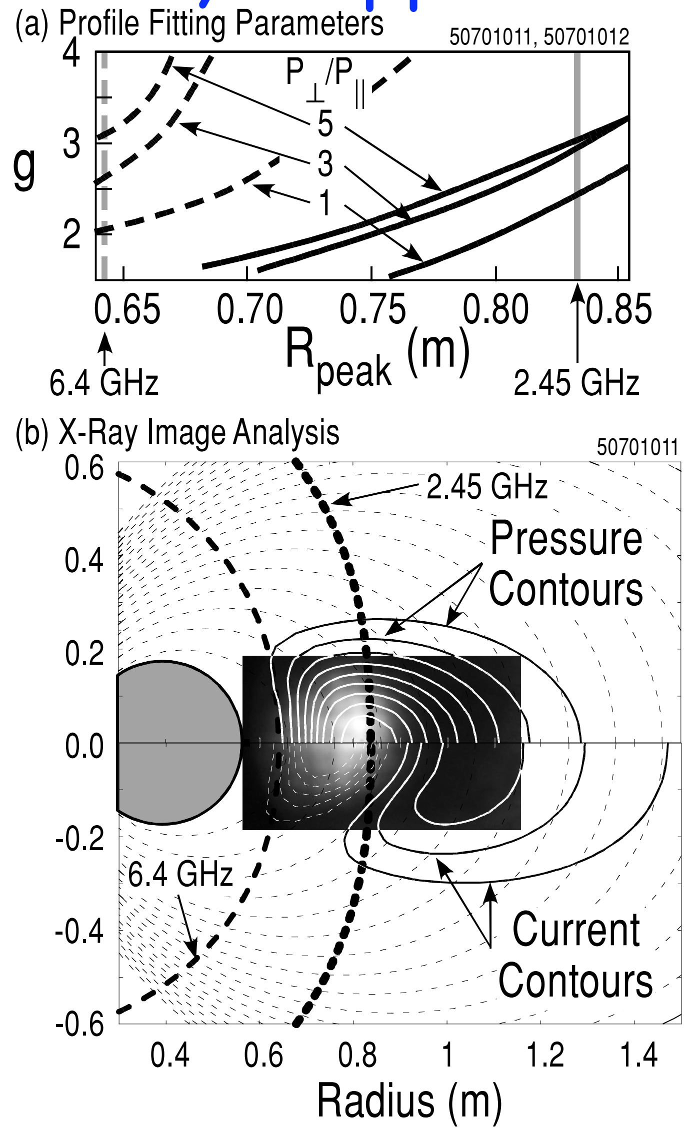


Fig. 1. Contours of the reconstructed pressure profiles superimposed onto the X-ray images measured during (top) 2.45 GHz heating and (bottom) 6.4 GHz heating.



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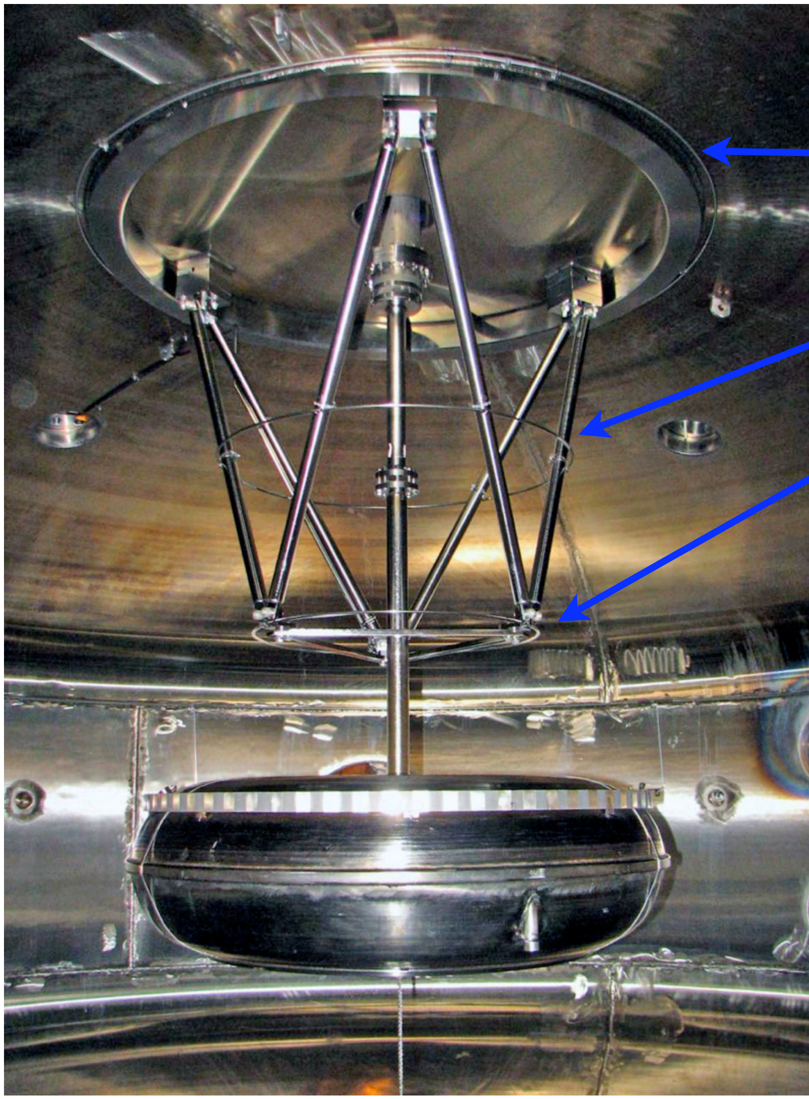
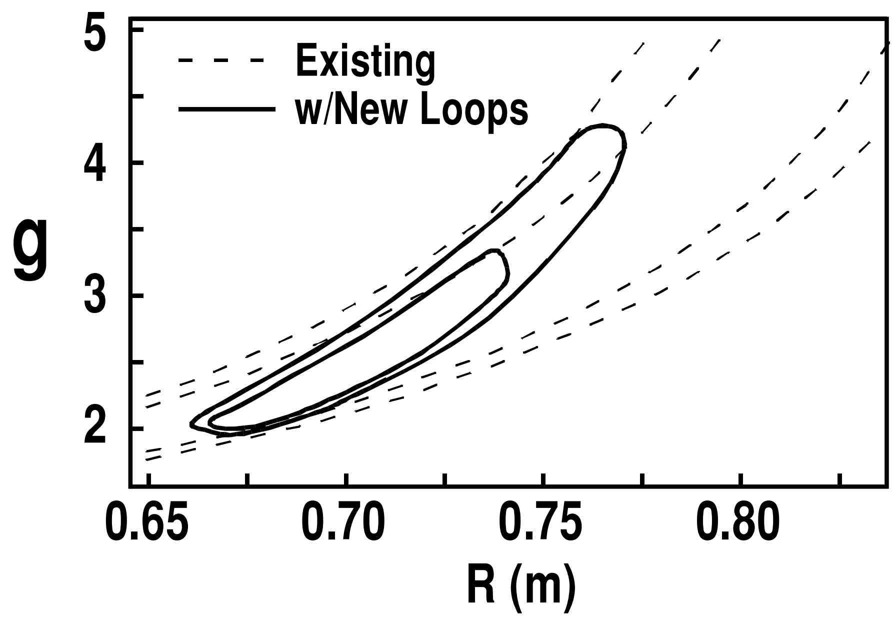
Newly Installed Internal Flux Loops Couple Better to Plasma Currents



Flux Loop 10
Flux Loop 11
Flux Loop 12

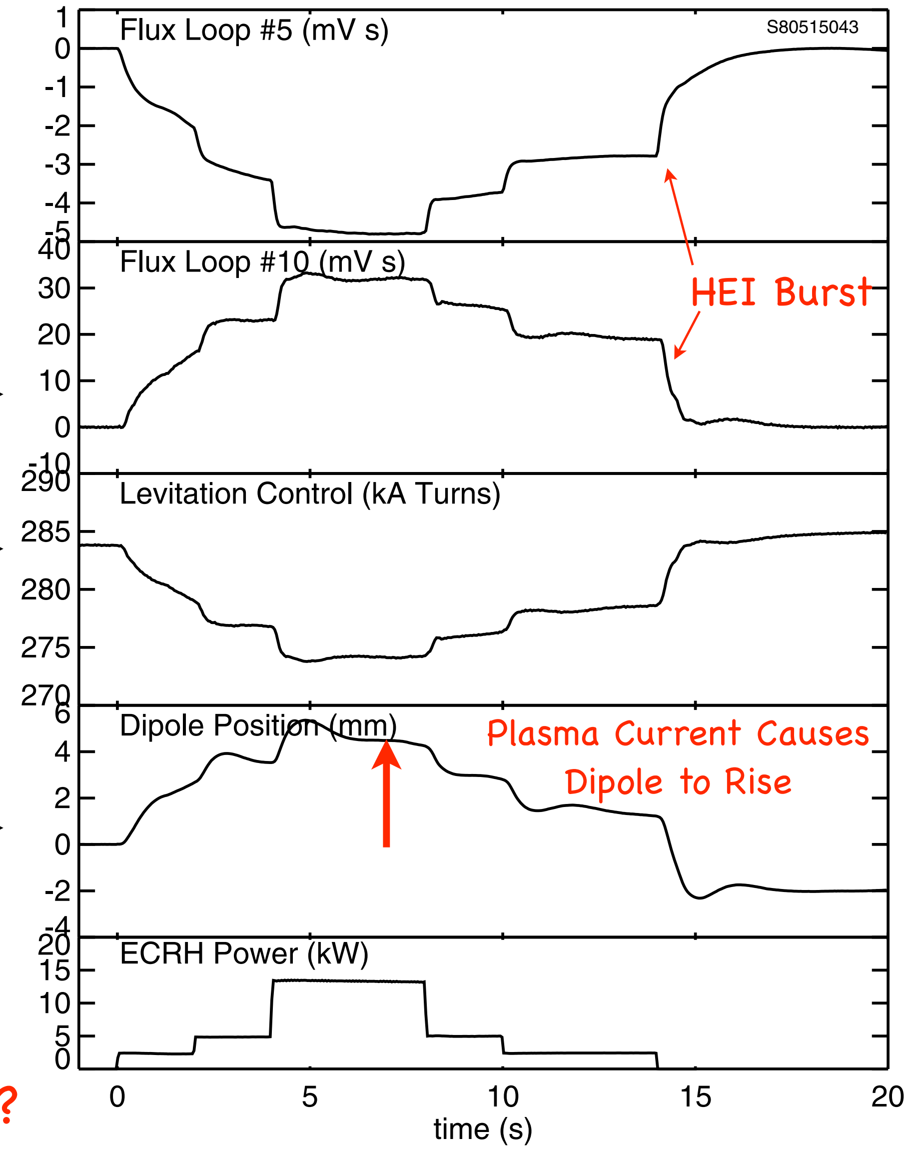


(2) "Self Calibration"

- With a levitated dipole, flux loops respond to control fields
- Control coil only: mutuals and induced eddy currents
- Dipole vertical displacement: mutuals

Typical Levitation

- External flux loop
- Internal flux loop
- ≈ 10 kA·turn variation of levitation control
- ≈ 4 mm motion of 1.1 MA·turn dipole

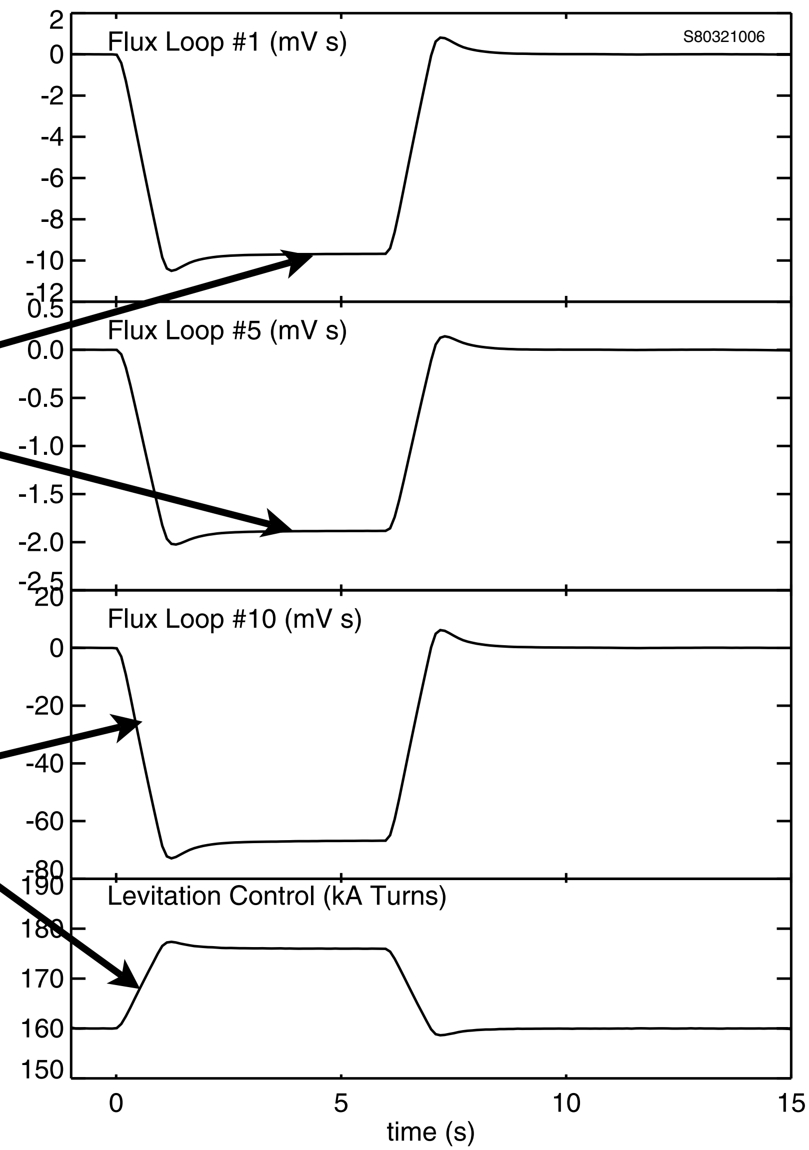


Question: What is the plasma contribution to magnetic signals?

Response from Levitation Control Coil

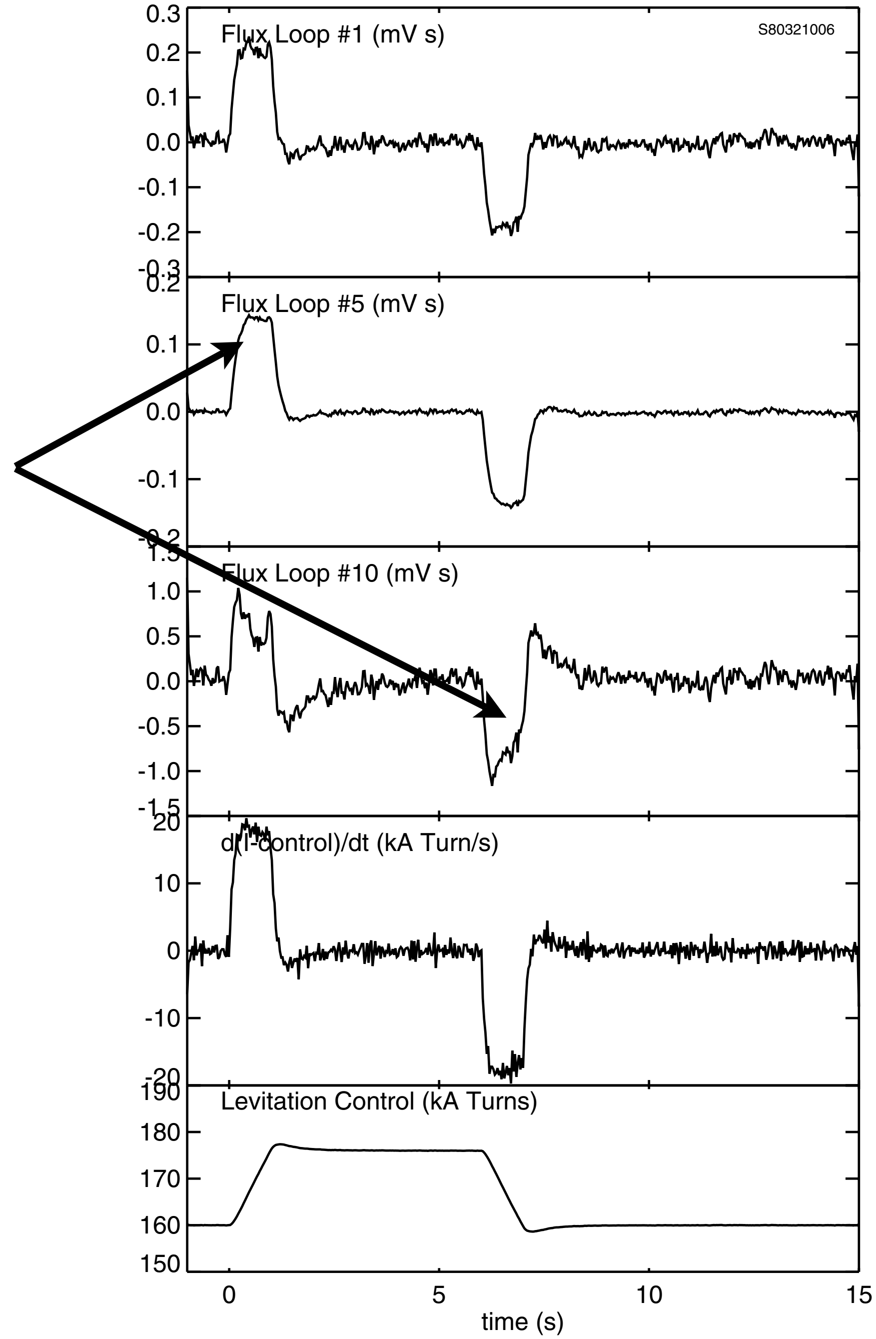
- Response from steady levitation current determines mutual inductance
- Response during constant current-ramp drives a constant eddy current

No plasma; No dipole.



Induced Eddy Currents

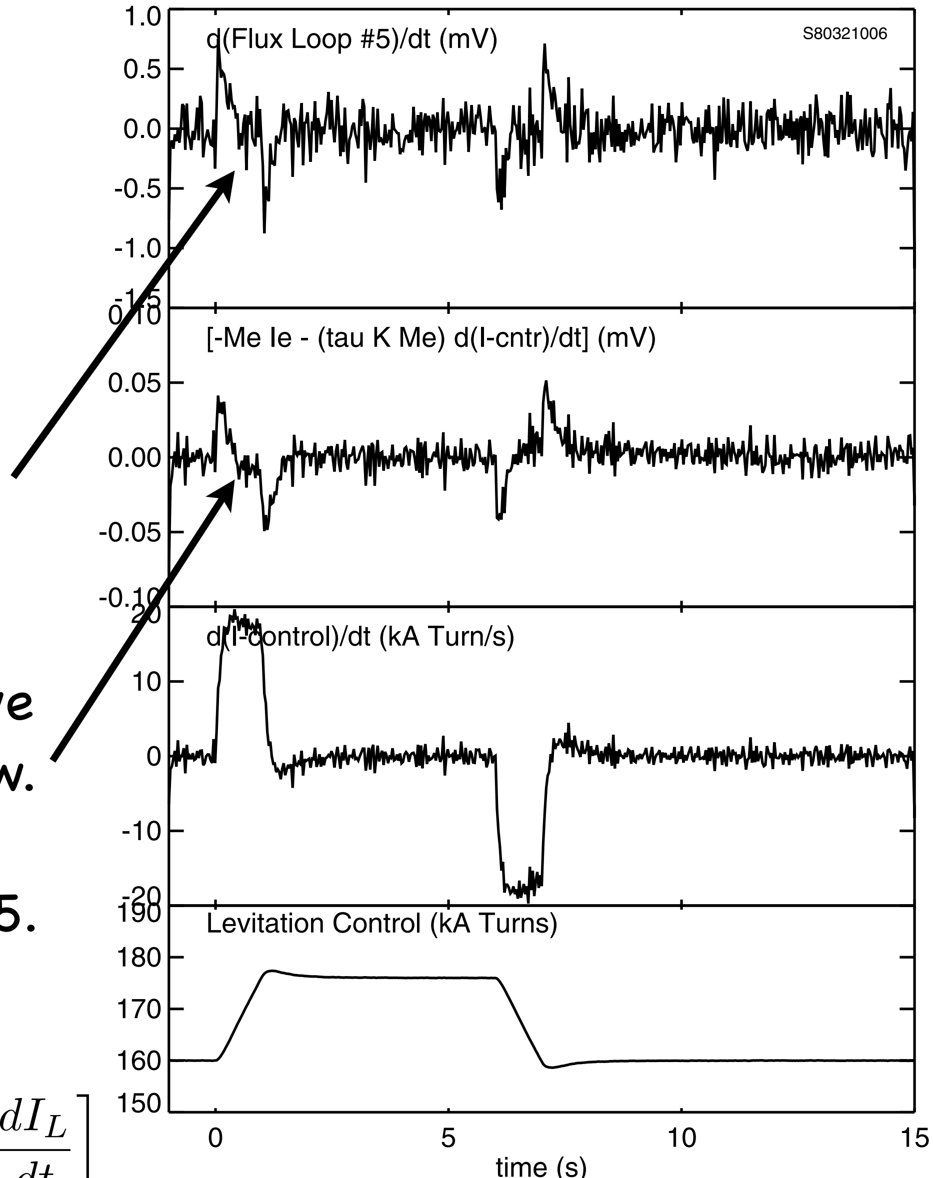
- Constant control current ramp drives a constant eddy current
- We need to find the **mutual** between eddy current and detector & the **wall eddy decay time, τ_W**
- The ratio of eddy current pick-up, $M_E I_E$, to dI_L/dt is equal to $\tau_W M_E K_{LE}$



Eddy Decay Time (τ_w)

- Take the numerical derivative of $(M_E I_E)$
- Find the ratio of this derivative to the eddy-drive shown below.
- $\tau_w \approx 71$ msec for Flux Loop #5.

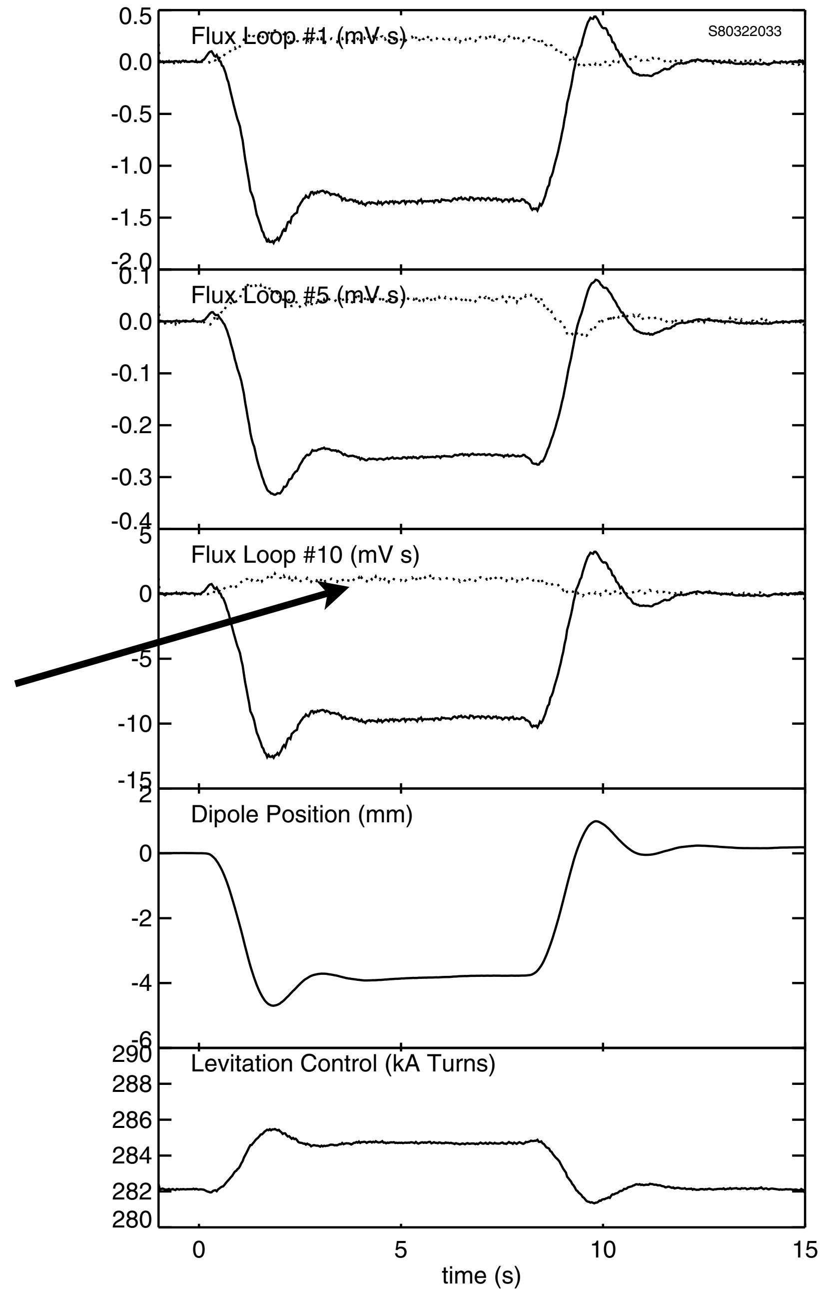
$$\frac{dM_E I_E}{dt} = \frac{1}{\tau_w}\left[-(M_E I_E) - (\tau_w K_L M_E)\frac{dI_L}{dt}\right]$$



Dipole Vertical "Jog"

- Without plasma, program a vertical displacement of dipole.
- After subtracting direct response from control coil, determine the response due to δz.
- For Flux Loop #5, 0.011 mV·s/mm

No plasma; Yes dipole.



Measured Coupling Coefficients

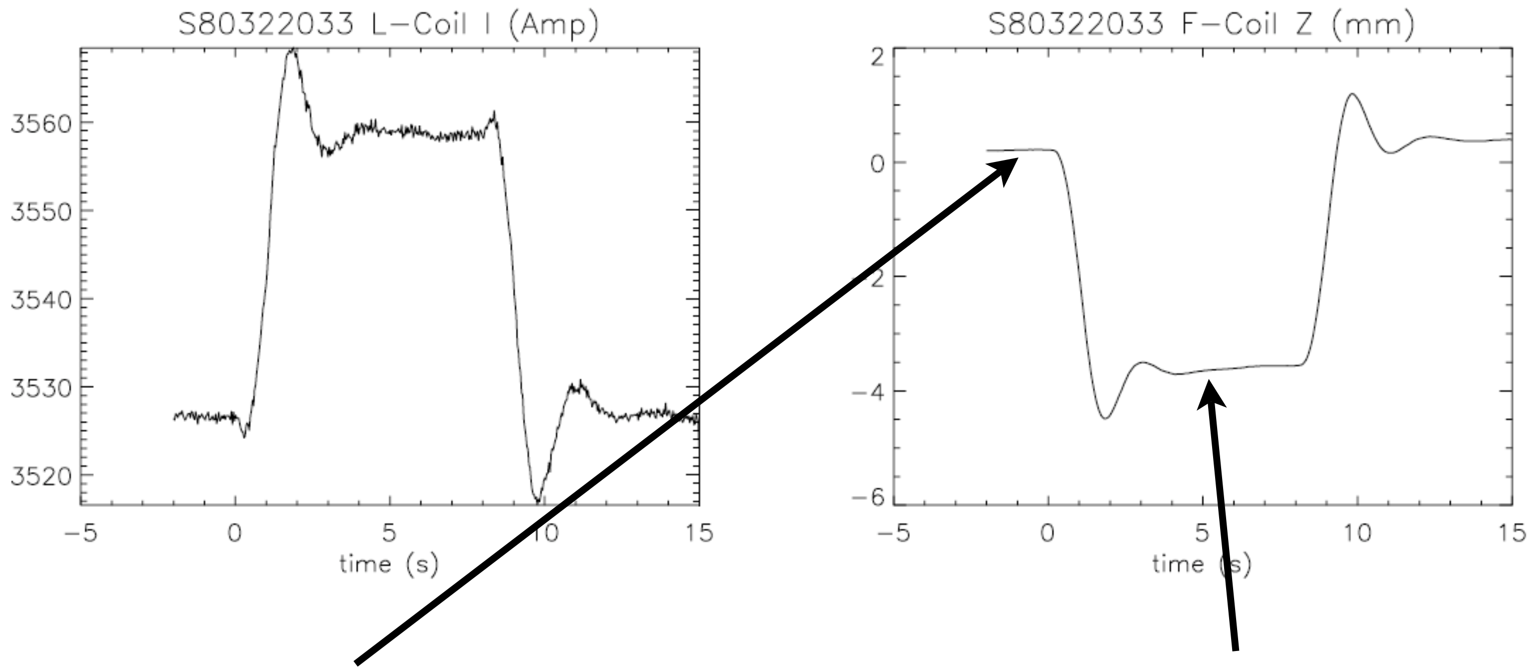
Flux Num	M_L (µH)	$K_L M_E$ (µH)	τ_W (ms)	G_Z (mV·s/mm)
1	48.4	42.0	21.7	-51.4
2	42.1	36.9	23.3	-59.8
3	35.4	33.5	26.4	-78.7
4	12.5	11.9	58.2	-30.5
5	9.41	8.37	71.3	-9.97
6	11.3	10.8	60.5	-30.2
7	1.64	1.11	148.5	85.9
8	1.15	1.02	118.8	79.9
9	0.584	0.406	167.3	56.8
10 (5 turns)	334.5	177.8	14.9	-245.4
11 (10 turns)	79.5	66.8	14.2	-1307
13 (30 turns)	14.4	10.2	25.7	-13,960
14 (50 turns)	5.94	5.79	25.1	-11,710

(3) How Much Dipole Current?

- Dipole current must be known for equilibrium reconstruction. **We "measure" dipole current using gravitational force balance.**
- Measured weight of dipole is 565 kg
- Control current required for levitation is
- Measured dipole position ($z \equiv 0$) gives dipole current of 1.116 MA·turn

F-Coil Charge

$$m_F g = -\frac{dM_{FL}}{dz}\left(I_F(0) - M_{FL} I_{lev}/L_F\right) I_{lev}$$

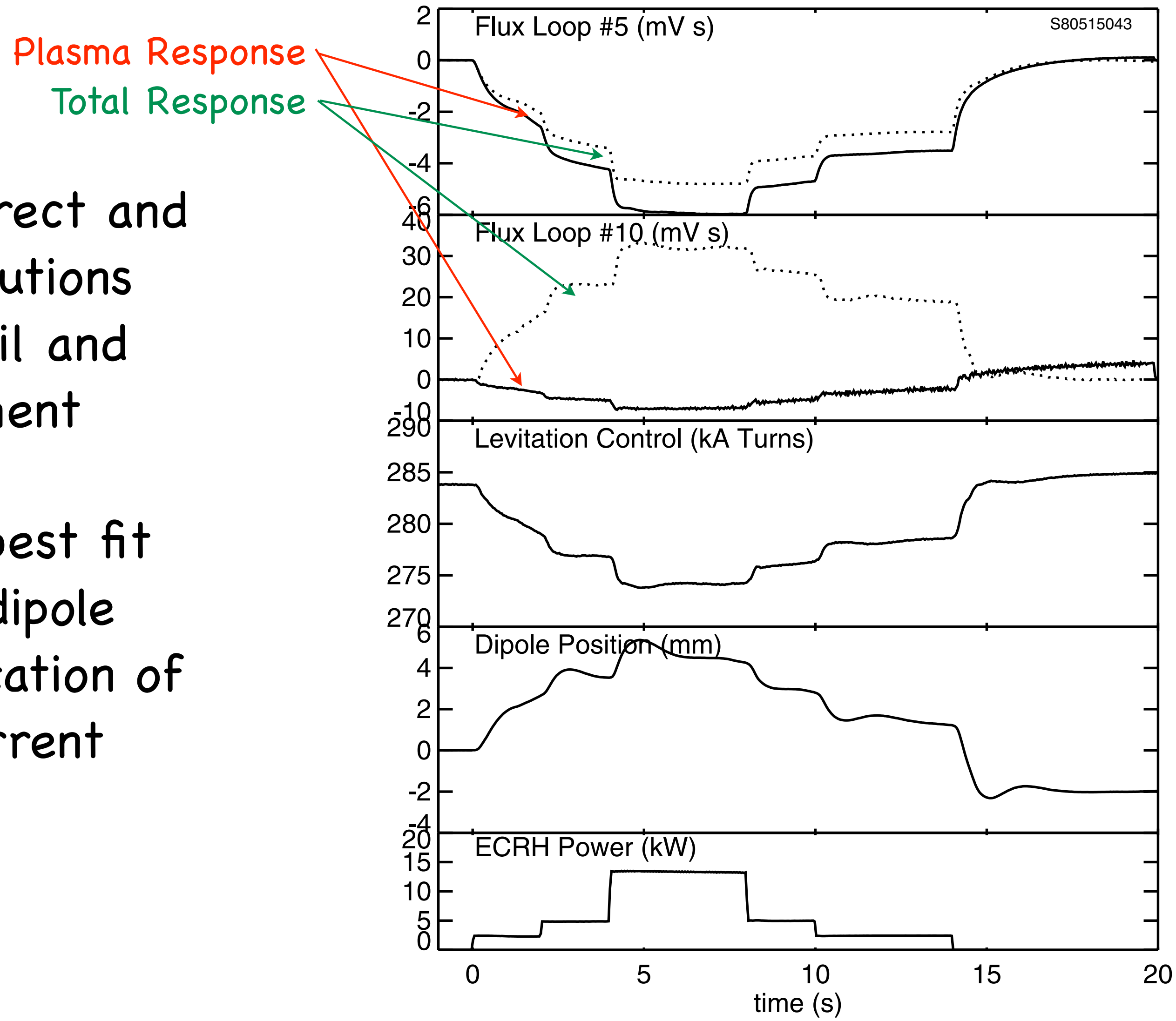


1.559 kA w M = 565 kg   1.558 kA w M = 565 kg

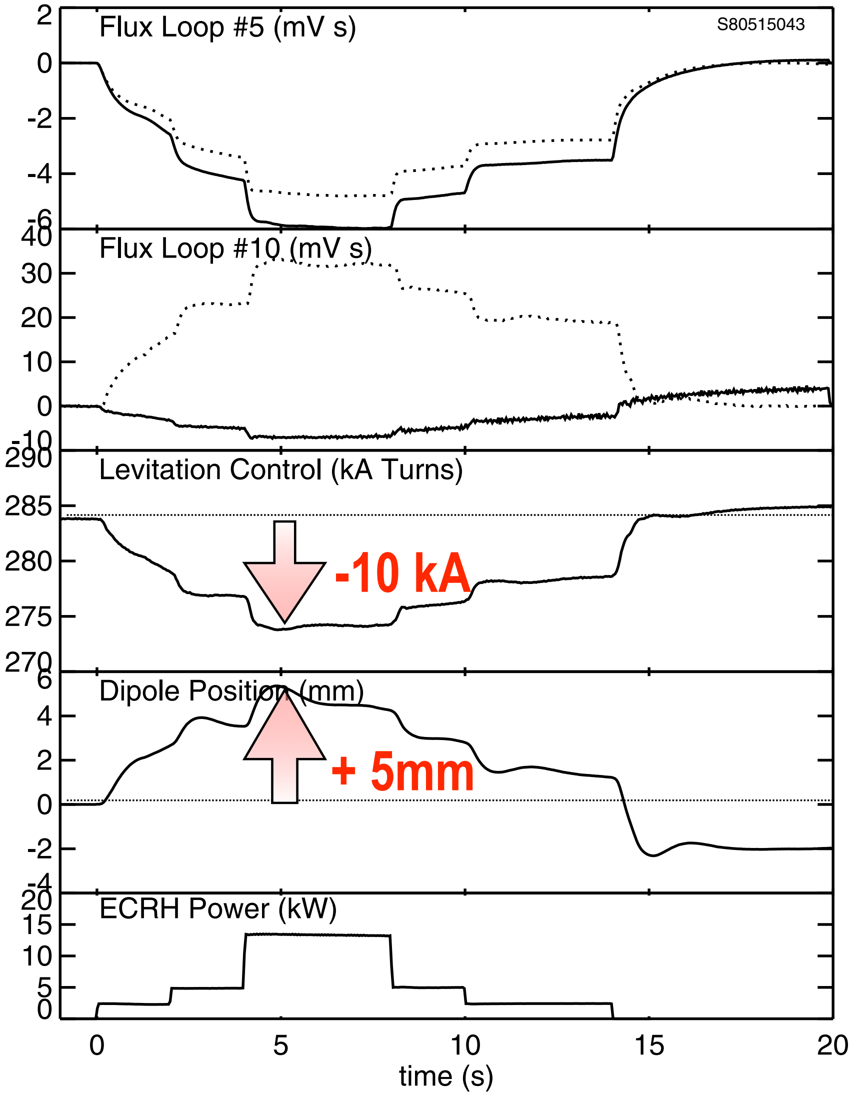
1.116 MA·turns Charge

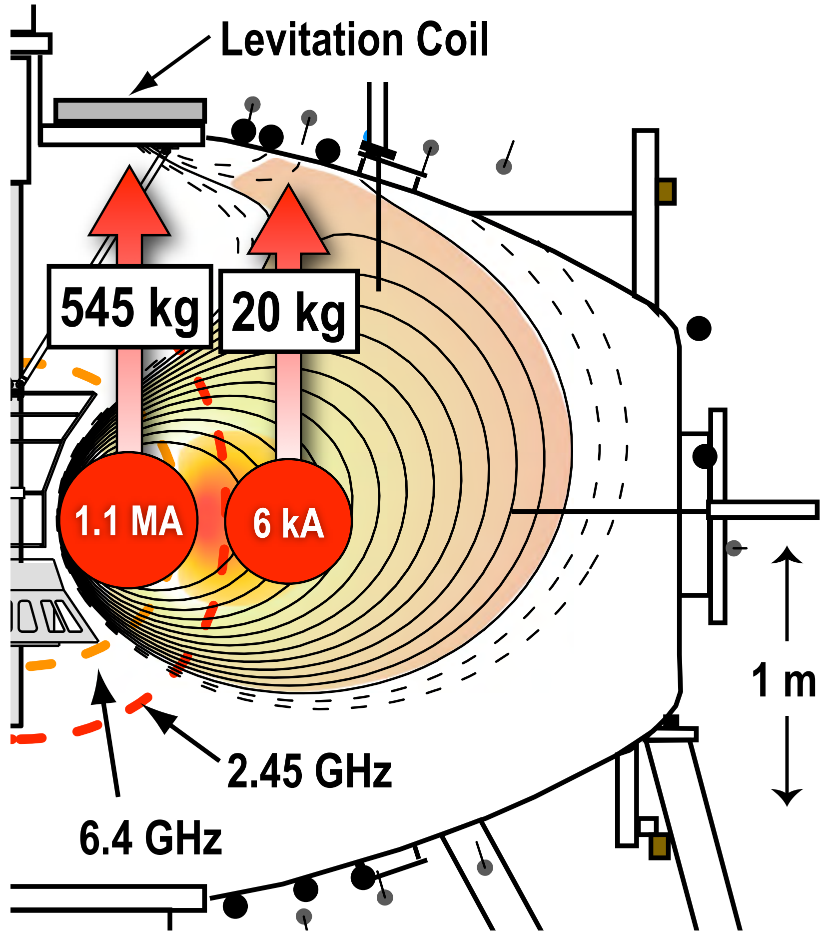
(4) How Much Plasma Current?

- Compute the direct and induced contributions from control coil and dipole displacement
- Least-squares best fit to find plasma dipole moment and location of diamagnetic current profile



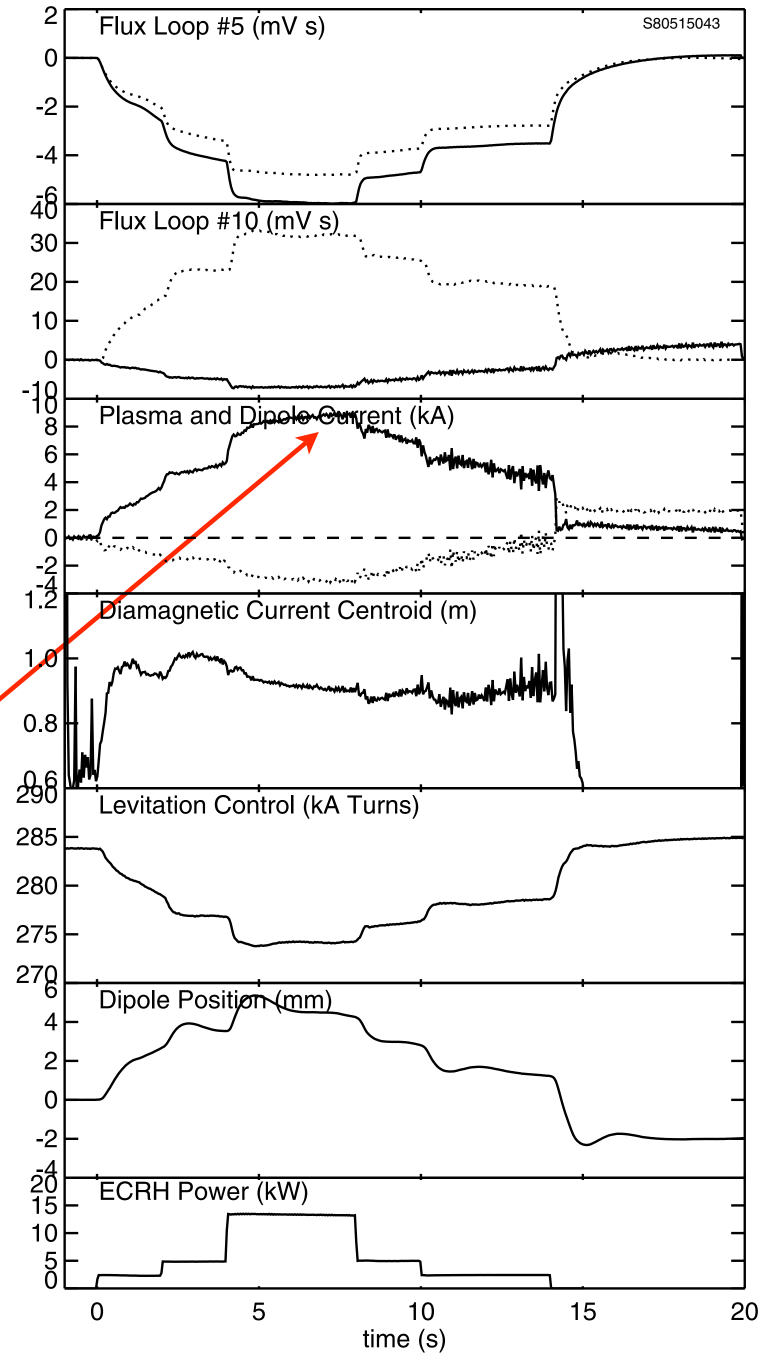
Higher Power ECRH and Levitation Produces Peak β ~ 35%





(4) How Much Plasma Current?

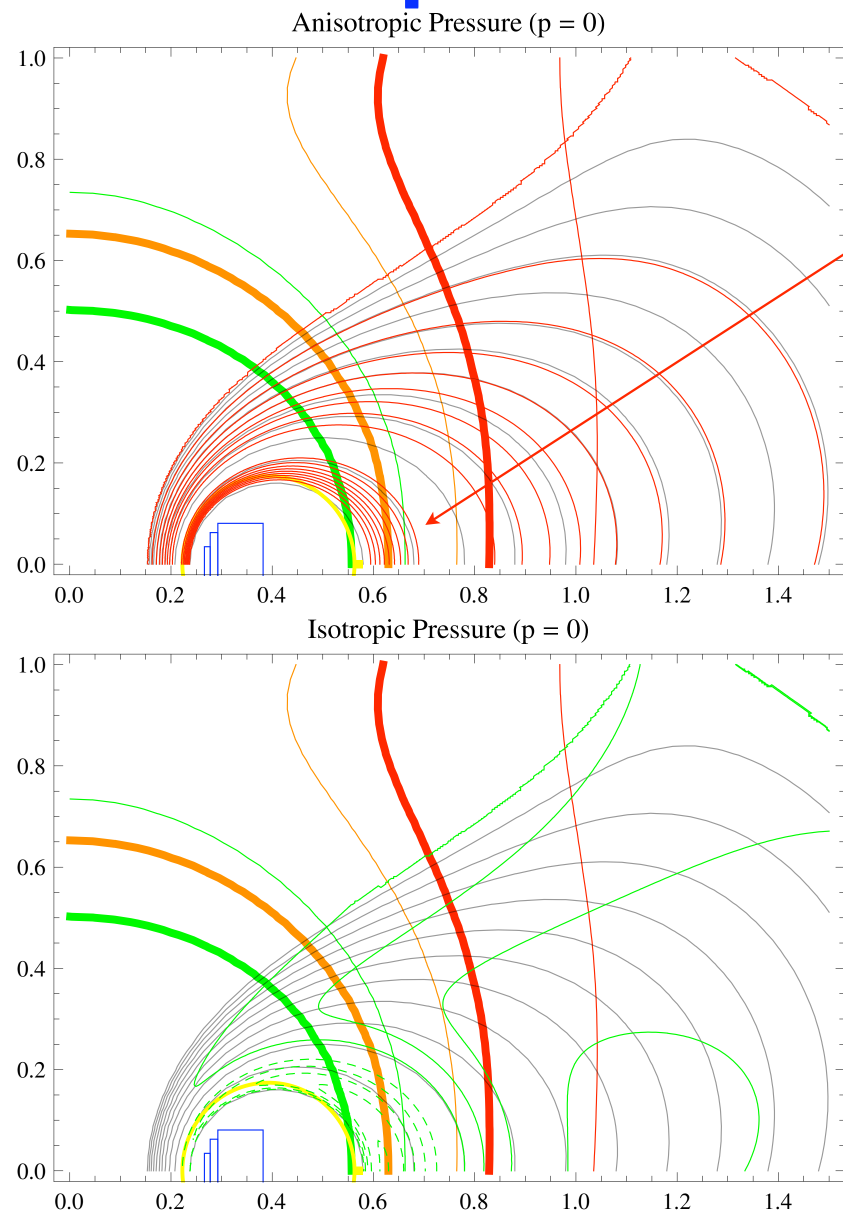
- Control-coil pickup is large for coils located nearby, at top of vessel
- High-power levitated discharges have large diamagnetic currents...
- $I_p \approx 9$ kA, 3×larger than previous!!
- Plasma stored energy more than 1 kJ



(5) Plasma Equilibrium with Levitated Dipole

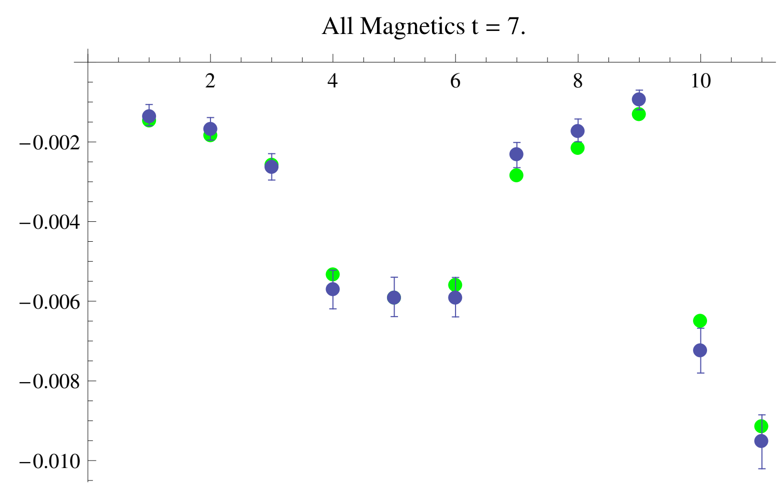
- First reconstructions with levitated dipole show **best fit profiles are isotropic**
- Plasma volume is 40% **smaller** (less stored energy)
- Best fit isotropic profile is **broad** for full heating power example: 10 GHz + 6.4 GHz + 2.45 GHz

Example Reconstruction



Peak Pressure at Innermost Closed Field Line

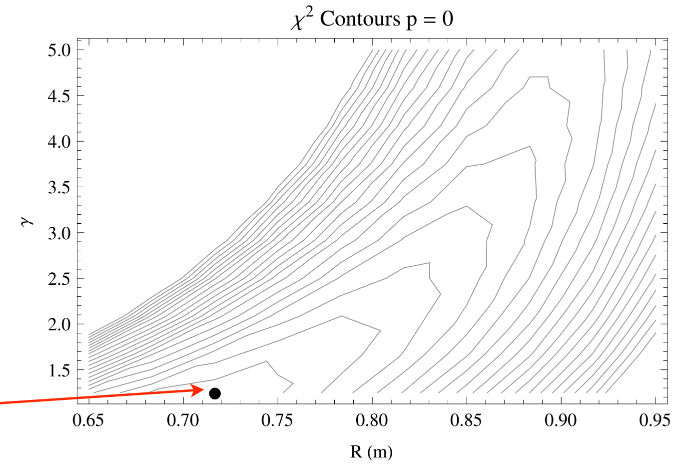
Parameter	Fit Value
χ^2	6.33824
Ip	-6581.13
δIf	1368.44
p	0
P(perp)/P(||)	1
R(peak)	0.75
γ	1.25
γ/(5/3)	0.75
Press(Rpeak)	139.12
J Centroid	1.18396
Moment (A m^2)	-9808.07
Max Perp β	0.415474
Perp β(Rpeak)	0.0237723
Avg Perp β	0.147046
Plasma Volume	11.4694
Energy (J)	736.583
E/Ip (J/kA)	-111.923

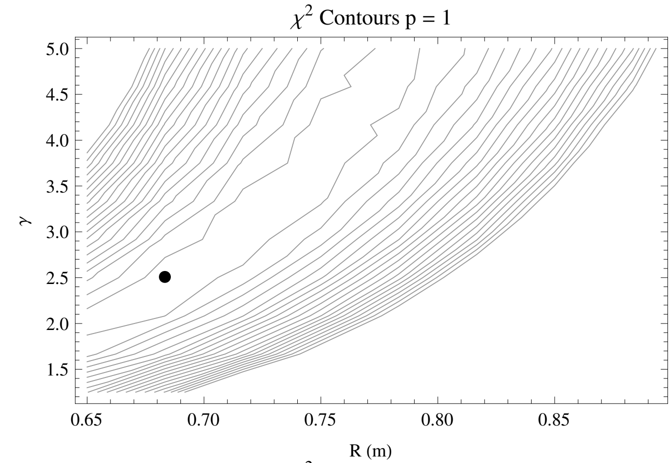


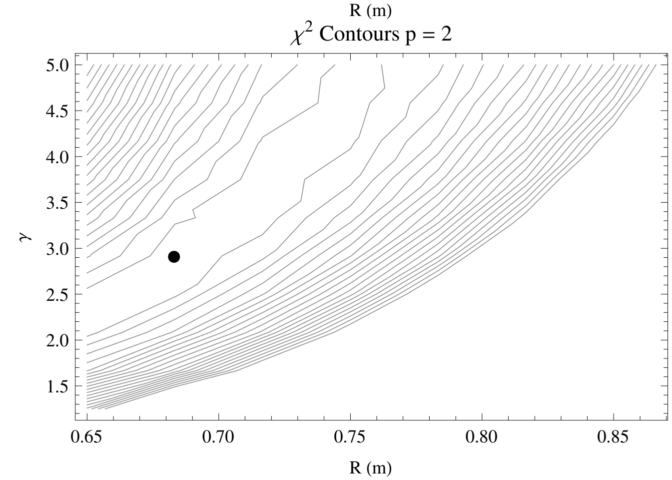
Best Fits w/Anisotropy

Isotropic!

Parameter	Fit Value	Fit Value	Fit Value
χ^2	6.33824	10.6887	11.8343
Ip	-6581.13	-6372.26	-6408.54
δIf	1368.44	1453.31	1477.22
p	0	1	2
P(perp)/P(||)	1	3	5
R(peak)	0.75	0.75	0.75
γ	1.25	3.75	4.58333
γ/(5/3)	0.75	2.25	2.75
Press(Rpeak)	139.12	989.039	1697.85
J Centroid	1.18396	1.14247	1.12411
Moment (A m^2)	-9808.07	-8343.91	-7992.52
Max Perp β	0.415474	0.271267	0.416496
Perp β(Rpeak)	0.0237723	0.169003	0.290122
Avg Perp β	0.147046	0.0451995	0.0349198
Plasma Volume	11.4694	11.4694	11.4694
Energy (J)	736.583	651.66	669.293
E/Ip (J/kA)	-111.923	-102.265	-104.438







Remaining "To Do" List

- Finalize free-boundary equilibrium calculations. (Initial results finished...)
- Compare equilibria during supported and non-supported operation. **Plasma pressure appears isotropic during levitation!**
- Complete eddy-current structure modeling to improve accuracy
- Incorporate additional external and internal magnetic probes.

Summary

- Magnetic reconstruction of the plasma current during dipole levitation requires subtraction of direct and induced pick-up from control coils and dipole position.
- A "self-calibration" procedure using pre-programmed control currents is used to measure the coupling coefficients
- Plasma currents are measured to exceed 9 kA, representing stored energy greater than 1 kJ!!