Helium Catalyzed D-D Fusion in a Levitated Dipole

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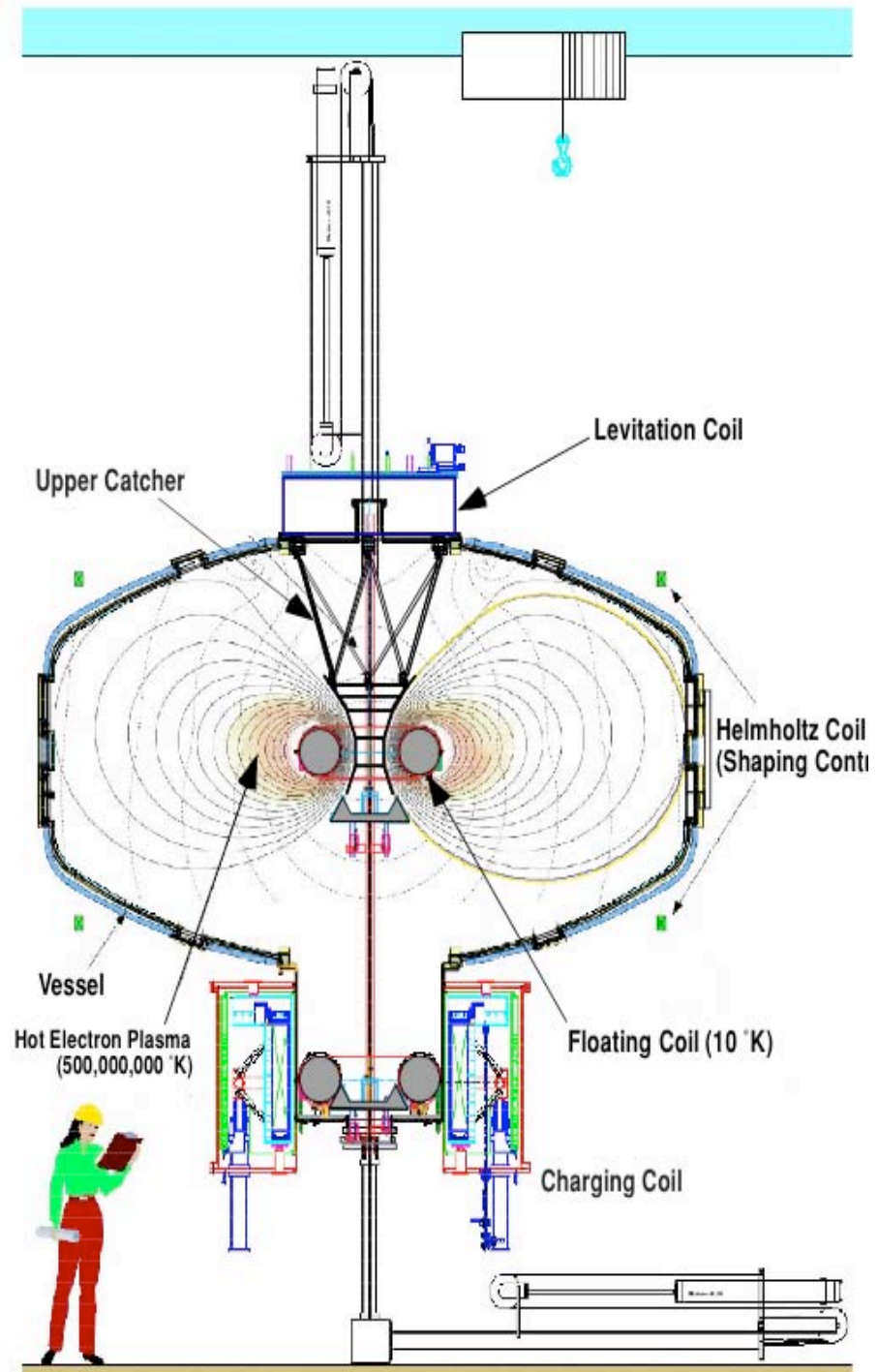*Columbia University*

Columbia University

Abstract

In plasma containing energetic electrons, two interacting collective modes, an MHD-like mode and a hot electron interchange (HEI) mode [N. A. Krall, Phys. Fluids, **9**, 820 (1966)], may be present. The linear stability of interchange modes in a z-pinch at arbitrary beta, including a bulk and hot electron species was recently studied [N. Krasheninnikova, P. J. Catto, Phys. Plasmas, **12**, 32101 (2005)]. Using the dispersion relation derived in this reference we show that when necessary conditions are satisfied the two modes may be present or absent in a closed-field line magnetic confinement geometry such as a hard core z-pinch or a dipole. The HEI instability and the MHD-like centrifugally-driven mode have been studied previously [B. Levitt, et al., Phys. Plasmas, **9**, 2507 (2002), and **12**, 055703 (2005).], including a comparison between the measured mode structure and the predictions of a global low-beta simulation. The radial eigenmode is seen to effect the saturation level of the mode. In the Levitated Dipole Experiment electron cyclotron resonance heating produces high beta plasmas containing hot electrons, and instability observations will be discussed and compared with theoretical predictions.
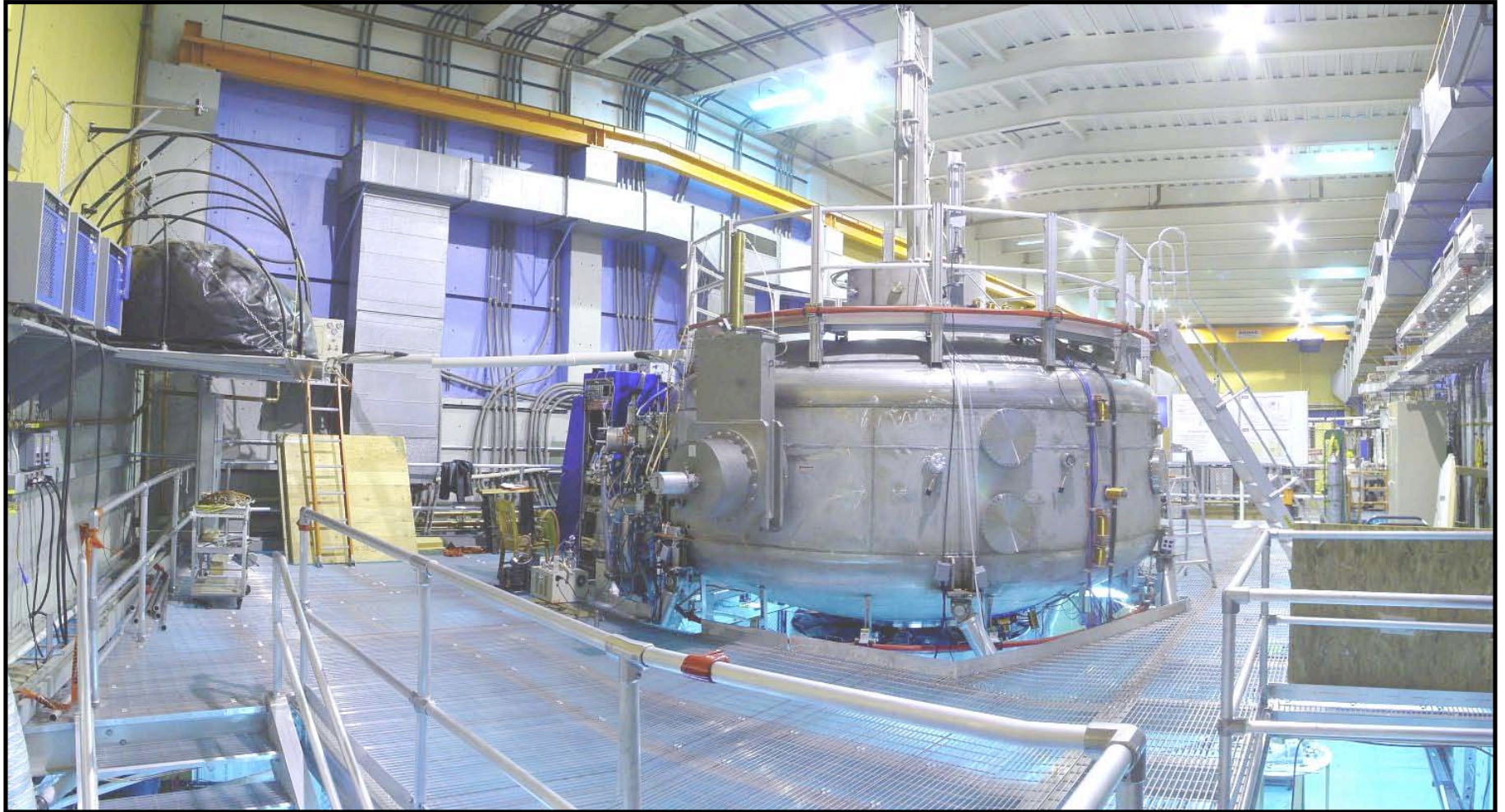
Outline

- ECRH in LDX

- Instability with hot electrons - linear theory

  ➢ Hot electron interchange (HEI) mode

  ➢ MHD-like background mode

- HEI: nonlinear simulations

  ➢ Chirping of HEI modes

  ➢ LDX Experimental observations

LDX Experiment

- LDX will explore stability & confinement limits in a dipole field.
  - ➢ Fusion concept for advanced fuels
- Will operate in either supported or levitated mode.
- Plasma created and maintained by ECH
  - ➢ Presently 2.45 and 6.4 GHz. Will add 10.6, 18 & 28 GHz.
  - ➢ Creates hot electron population ($T_{eh} > 50$KeV).
- In addition to MHD modes can be unstable to HEI and loss cone modes.

Levitation Coil

Upper Catcher

Helmholtz Coil (Shaping Contr

Vessel

Hot Electron Plasma (500,000,000 ˚K)

Floating Coil (10 ˚K)

Charging Coil

The Levitated Dipole Experiment (LDX)


MIT/Columbia Research Project

Compressibility can stabilize interchanges

- No compressibility:

"bad" κ & ∇B drifts causes
 charge separation ->

V_{ExB} increases perturbation

Plasma

Ions

electrons

Vacuum

E x B

B x
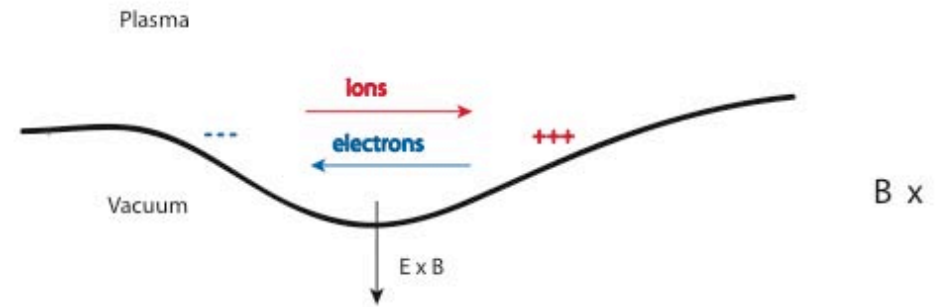
- With compressibility: as
 plasma moves downwards
 pressure decreases. For
 critical gradient there is no
 charge buildup

Density gradient.
Compressibility: Density decreases as plasma moved downward.

Dense Plasma

Lower density plasma

E x B

B x
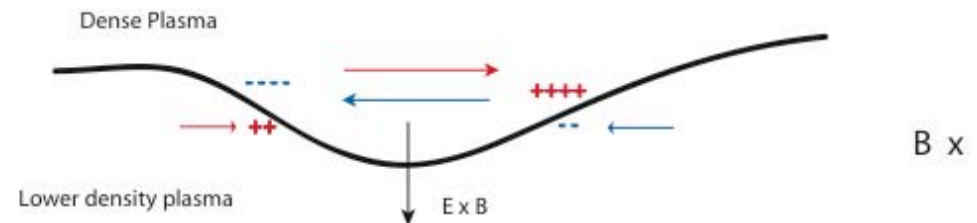
In bad curvature pressure
 gradient is limited to $\quad -\dfrac{d\ln p}{d\ln V} < \gamma \qquad V = \oint dl/B$

- 6

Review: MHD Results

- Equilibria exist at high-β.

- Marginally stable for interchange modes satisfy adiabaticity condition at all β.

  ➢ M.N. Rosenbluth and Longmire, *Ann. Phys.* **1** (1957) 120.

$$\delta(pV^{\gamma}) \equiv \delta(S) = 0, \text{ where } V \equiv \oint \frac{dl}{B}, \ \gamma = \frac{5}{3}$$

- Stable to ideal MHD ballooning when interchange stable

- No Magnetic Shear -> Convective cells develop when interchange stability violated

  ➢ Keeps ∇p near critical value.

  ➢ For marginal profiles, convective cells will convect particles but not energy.

  ◆ Leads to have low τ_p with high τ_E . This property makes LDX particularly interesting for advanced fuels.

LDX has two plasma regions

Bad Curvature region (between pressure peak & vacuum vessel)

- MHD: stable to interchange when $\delta(pV^\gamma)>0$, $V = \oint dl/B$
  $p_{core}/p_{edge} < (V_{edge}/V_{core})^\gamma \sim 10^3$: want large vacuum chamber
  - ➤ MHD stability from field bending and not grad-B -> $\beta \sim 1$
  - ➤ Unstable interchange modes evolve into convective cells
- Ballooning modes stable when interchange stable
- Weak resistive mode at high β ($\gamma \sim \gamma_{res}$ but no $\gamma \sim \gamma_{res}^{1/3} \gamma_A^{1/3}$ mode)
- Drift modes: electrostatic "entropy" mode when $\delta(nV)>0$ ($\eta<2/3$), i.e.
  $n_{core}/n_{edge} < (V_{edge}/V_{core})$

Good Curvature region (between floating coil and pressure peak)

- "Entropy" (drift) mode can be unstable when $grad(n_e)<0$

Convective Cells in Dipole

- **Convective cells can form in closed-field-line topology**
  - ➤ Field lines charge up -> R-ϕ convective flows
  - ➤ 2-D nonlinear cascade leads to large scale vortices
  - ➤ Cells circulate particles between core and edge
    - ◆ No energy flow when pV^{γ}=constant, (i.e. $p'=p'_{crit}$).
    - ◆ When $p'>p'_{crit}$ cells get non-local energy transport. Only transport sufficient energy transport to maintain $p' \gtrsim p'_{crit}$.
    - ◆ Selective ``pumping'' at plasma edge can remove tritium.
  - ➤ Non-linear calculations use reduced MHD or PIC

wall
R
Pressure peak
coil
2.0
1.5
1.0
0.5
Potential contours ϕ
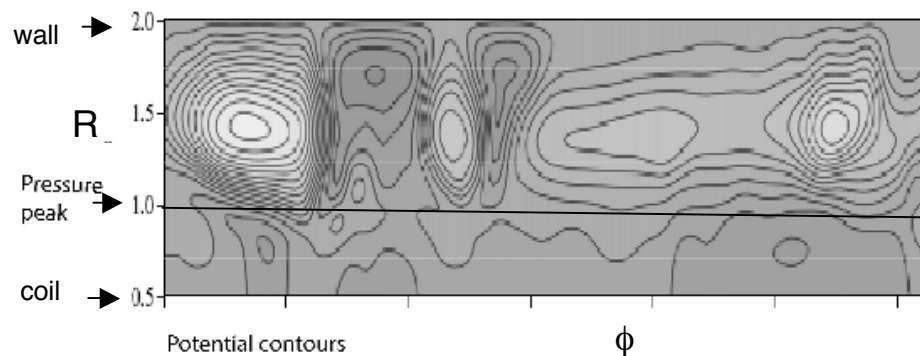
Reduced MHD: Pastukhov, Chudin, Pl Physics 27 (2001) 907.

PIC: Tonge, Leboeuf, Huang, Dawson, submitted to Phys Pl. (2003).

Systems with large compressibility

In a closed field line system with a strong field line curvature two basic properties become important in deterrmining the stability of interchange modes. When the plasma pressure varies as

$$\frac{\partial}{\partial \psi}(pV^\gamma) = 0 \quad \rightarrow \quad d_p \equiv -\frac{dln\ p}{dln\ V} = \gamma \tag{1}$$

with ψ the flux co-ordinate, $\gamma = 5/3$ and $V = \partial(Vol)/\partial\psi = \oint d\ell/B$, the differential flux tube volume, an exchange of flux tubes will not change the pressure profile. Similarly when

$$\frac{\partial}{\partial \psi}(n_e V) = 0 \quad \rightarrow \quad d_n \equiv -\frac{dln\ n_e}{dln\ V} = 1 \tag{2}$$

an exchange of flux tubes does not change the density profile.

ECH in dipole

- ECH heats most strongly where mod-B tangent to field
- Quasi-linear diffusion along ECH characteristics creates radially localized and anisotropic hot electrons.
  - ➢ Radially localization yields instability at $f \sim \Omega_{de}$ (conserving μ and J) yielding X-field transport: HEI
  - ➢ Anisotropic distribution yields instability at $f \sim \Omega_{ce}$ pitch angle scatter (and loss to supports).

ECRH creates a hot electron species; $T_{eh} \gg T_b$

- **Stability similar to EBT (bumpy torus):**
  - MHD-like background mode and kinetic hot electron interchange can both be present.
  - EBT symbiosis -> fragile stability: Background stabilized by diamagnetic well of hot electrons. Hot electron stability requires $n_{eh}/n_b < N_{crit} \sim 0.2$

- **In Dipole, MHD mode stabilized by compressibility**
  - MHD-like mode would lead to convective motion of background which tends to create $n_{core}/n_{edge} \sim V_{edge}/V_{core}$, $p_{core}/p_{edge} \sim (V_{edge}/V_{core})^\gamma$, i.e. to centrally peaked n_b & p.
  - In LDX shaping (Helmholtz) coils can vary V_{edge}/V_{core}

  (Note: ECH is not intrinsic to the dipole concept as it is for EBT)

Hot Electron Interchange Mode (HEI)

- Plasma containing an electron species can be subject to two unstable modes (EBT, ECH mirrors):

  ➢ MHD-like mode feeds on background plasma free energy, $f \sim f_{MHD}$

  ➢ HEI feds on hot electron density gradient, $f \sim f_{dh}$

- HEI unstable when density gradient exceeds critical value:

$$-\frac{d \ln n_{eh}}{d \ln V} < 1 + \frac{m_\perp^2}{6} \frac{\omega_{dh}}{\omega_{ci}} \frac{n_e}{n_{eh}} \quad with \ V = \oint \frac{d\ell}{B}$$
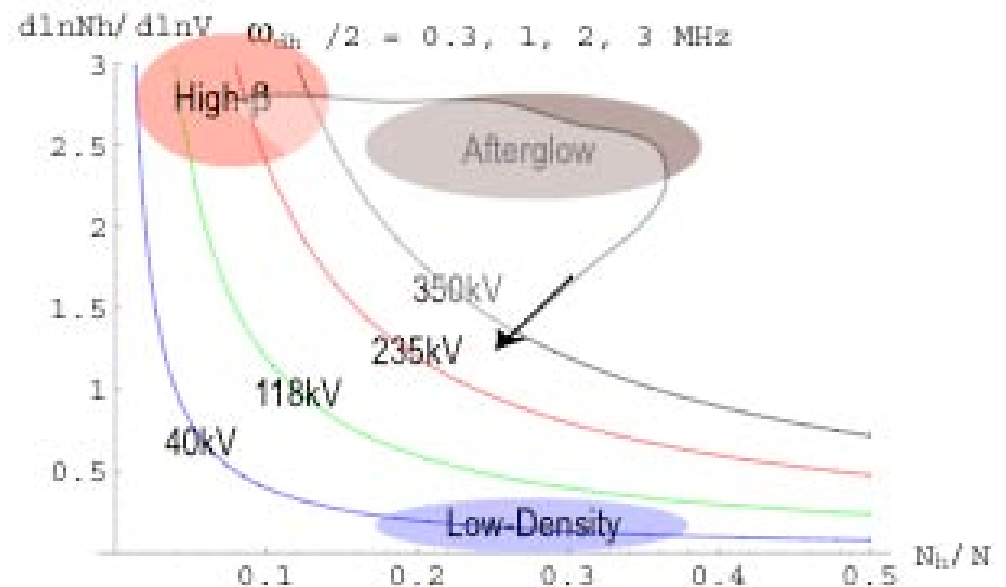
Ref: Krall PF 9 (66), Berk PF 13 (70)

Stabilized by background plasma when $\frac{n_{eh}}{n_e} < \alpha, \ 0.1 < \alpha < 1$

- Background mode stability requires $-\frac{d \ln p_b}{d \ln V} < \gamma$

- (In presence of loss cone plasma can also generate whistler instability at f_{ce}).

LDX exhibits three stability regimes

HEI creates a 3-dimensional space in hot electron fraction, density gradient and mean electron energy.

- Low density regime: high n_{eh}/n_e and low T_{eh} limits $d\ln n_{eh}/d\ln V$.

  ➢ ECH maintains peaked & unstable profiles

- High β regime: Very hot electrons with low n_{eh}/n_e.

  ➢ During bursts spreading can stabilize mode

- After glow: Loss of cold plasma yields high n_{eh}/n_e.

  ➢ No ECH so spreading can stabilize mode.

Hot Electron Interchange (HEI) mode

References:

- Krall PF 9 (66), Berk PF 13 (70)
- CTX studies (Low β dipole):

  Warren, Mauel, PRL 74(95), Maslovsky, Leavitt, Mauel

  PRL90 (2003), …
- Krasheninnikova, Catto, PF 12 (05)32101. Hard-core z-pinch. Hot electrons in finite pressure fluid background background at arbitrary β.

- HEI requires $-d\ln n_{eh}/d\ln V > 1$ and $n_{eh}/n_e > N_{crit}$
- MHD-like mode requires $-d\ln p_b/d\ln V > \gamma$
    - **In large vacuum chamber of LDX, n_e and T_{eh} gradients criteria can relax.**
    - **For low n_b, the ratio (n_{eh}/n_e) not effected by plasma spreading and HEI is more dangerous**
- Several parameters determine stability
- LDX observes HEI and MHD-like mode in high β plasma.

High β, Z-pinch Dispersion Relation

- Arbitrary β, hard-core z-pinch dispersion relation. Maxwell-hot electrons (Krasheninnikova, Catto, PF 12 (05)32101)

$$\left(1 + \frac{\beta_b \gamma}{2} - \frac{r\beta_h p'_{0h}}{2p_{0h}}I\right)\left(-\frac{(\frac{dln\ p_b}{dlnV} + \gamma)\omega_{db}{}^2}{\omega^2} + b + \frac{n_{0h}T_e}{p_{0b}}\left(\frac{T_e G}{T_h} + \frac{(\frac{dln\ n_{eh}}{dlnV} + 1)\omega_{db}}{\omega}\right)\right)$$

$$+ \frac{\beta_b}{2}\left((1 - H)\frac{n_{0h}T_e}{p_{0b}} + \frac{(\frac{dln\ p_b}{dlnV} + \gamma)\omega_{db}}{\omega}\right)^2 = 0.$$

Notice appearance of $\dfrac{dln\ p_b}{dlnV} + \gamma$ and $\dfrac{dln\ n_{eh}}{dlnV} + 1$

- Limit of β_b, $T_{eb} \to 0$; n_{eb} finite

$$\frac{\omega}{\omega_{\kappa h}} = \frac{-2\frac{n_{eh}}{n_b}(1 + \frac{dlnn_{eh}}{dlnV})(1 - \frac{r\beta_h p'_{0h}}{2p_{0h}}I(\omega))}{2(\frac{n_{eh}}{n_b}G(\omega) + b\frac{T_h}{T_b})(1 - \frac{r\beta_h p'_{0h}}{2p_{0h}}I(\omega)) + \frac{n_{eh}\beta_h}{n_b}(1 - H(\omega))^2}$$

- G, H, I are complex functions that contain integrals over the hot electron distribution function.

Integrals over hot electron distribution function

The perturbed hot electron densities and currents, n_{1h} and J_{1hr} as a function of perturbed fields fields, ϕ and $B_{1\theta}$ are:

$$n_{1h}/n_{0h} = \frac{\phi}{T_h}G + \frac{B_{1\theta}}{B_0}H \quad and \quad (\mu_0 J_{ihr}) = \frac{e\phi}{2T_h}\beta_h H + \frac{B_{1\theta}}{B_0}\frac{r\beta_0 p'_{0h}}{2p_{0h}}I$$

With integrals over velocity ($t^2 = m_e v^2/2T_{eh}$) and pitch angle ($\lambda = v_{ll}/v$)

$$G = 1 + \frac{2}{\sqrt{\pi}\omega_{\kappa h}}\int_0^\infty dt\, e^{-t^2}(\omega - \omega_{*h}[1+\eta_h(t^2 - \frac{3}{2})]) \int_{-1}^1 \frac{d\lambda}{\hat{D}(s) - \omega/(\omega_{\kappa h}t^2)}$$
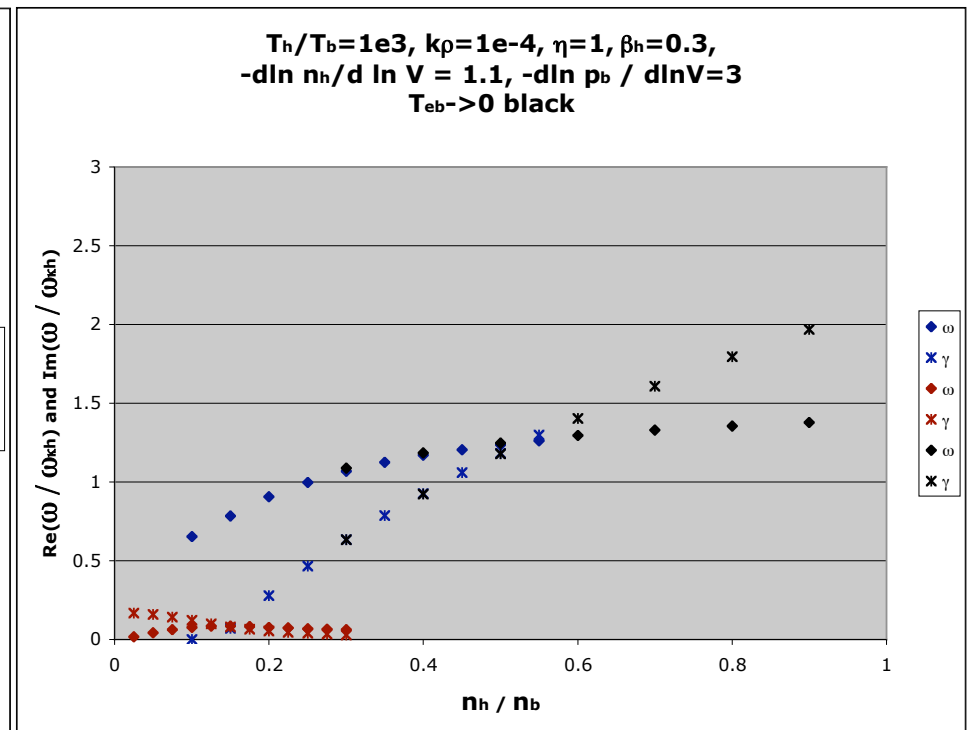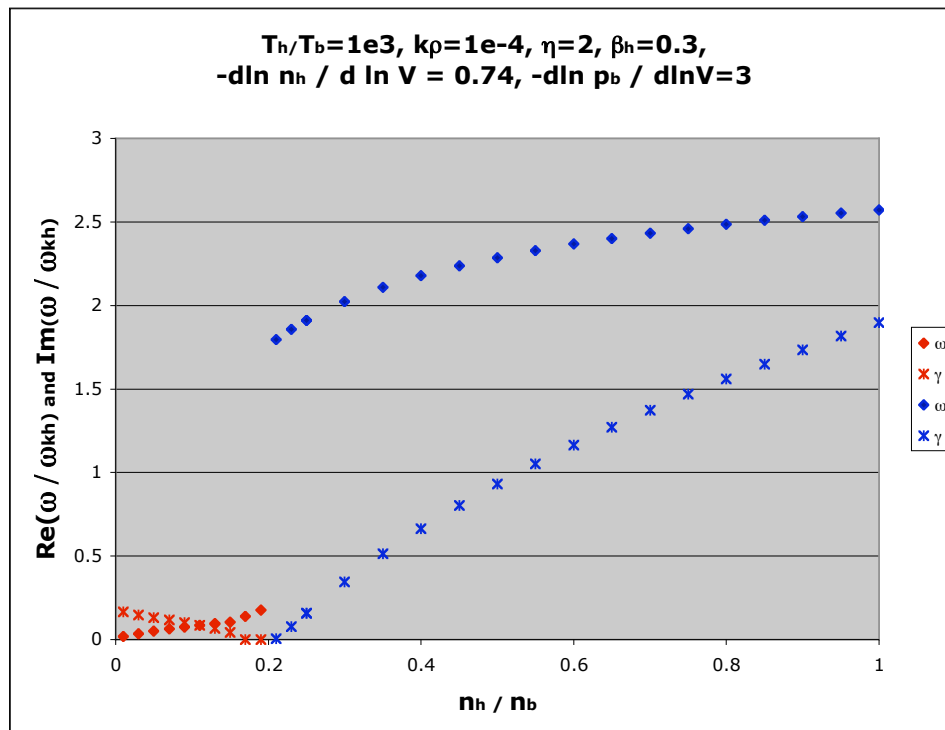
$$H = -\frac{2}{\sqrt{\pi}\omega_{\kappa h}}\int_0^\infty dt\, e^{-t^2}t^2(\omega - \omega_{*h}[1+\eta_h(t^2 - \frac{3}{2})]) \int_{-1}^1 \frac{d\lambda(1-\lambda^2)}{\hat{D}(s) - \omega/(\omega_{\kappa h}t^2)}$$

$$-\frac{r\beta_h p'_{0h}}{2p_{0h}}I = -\frac{\beta_h}{\sqrt{\pi}\omega_{\kappa h}}\int_0^\infty dt\, e^{-t^4}(\omega - \omega_{*h}[1+\eta_h(t^2 - \frac{3}{2})]) \int_{-1}^1 \frac{d\lambda(1-\lambda^2)^2}{\hat{D}(s) - \omega/(\omega_{\kappa h}t^2)}$$

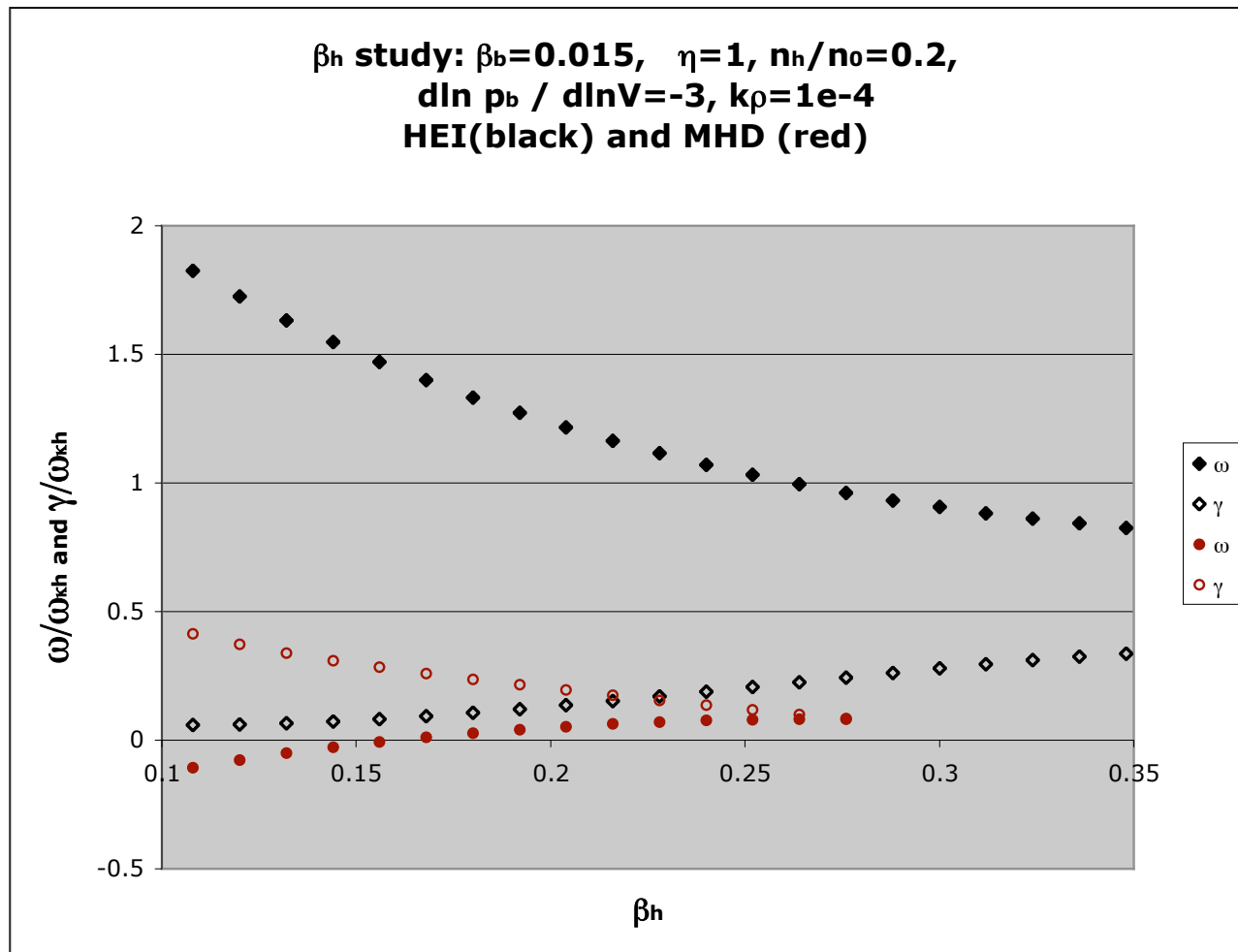The t-integrals are evaluated numerically using mathematica.

Parameter study

- Can observe both MHD and HEI modes
  - Depends on $\eta = d\ln T_{eh} / d\ln n_{eh}$
- For HEI key parameters are n_{eh}/n_b, T_{he}/T_b, η, $d\ln n_{eh}/d\ln V$
- For MHD mode key parameter is $d\ln p_b / d\ln V$

$T_h/T_b=1e3$, $k\rho=1e-4$, $\eta=2$, $\beta_h=0.3$, $-d\ln n_h / d\ln V = 0.74$, $-d\ln p_b / d\ln V=3$

$T_h/T_b=1e3$, $k\rho=1e-4$, $\eta=1$, $\beta_h=0.3$, $-d\ln n_h/d\ln V = 1.1$, $-d\ln p_b / d\ln V=3$
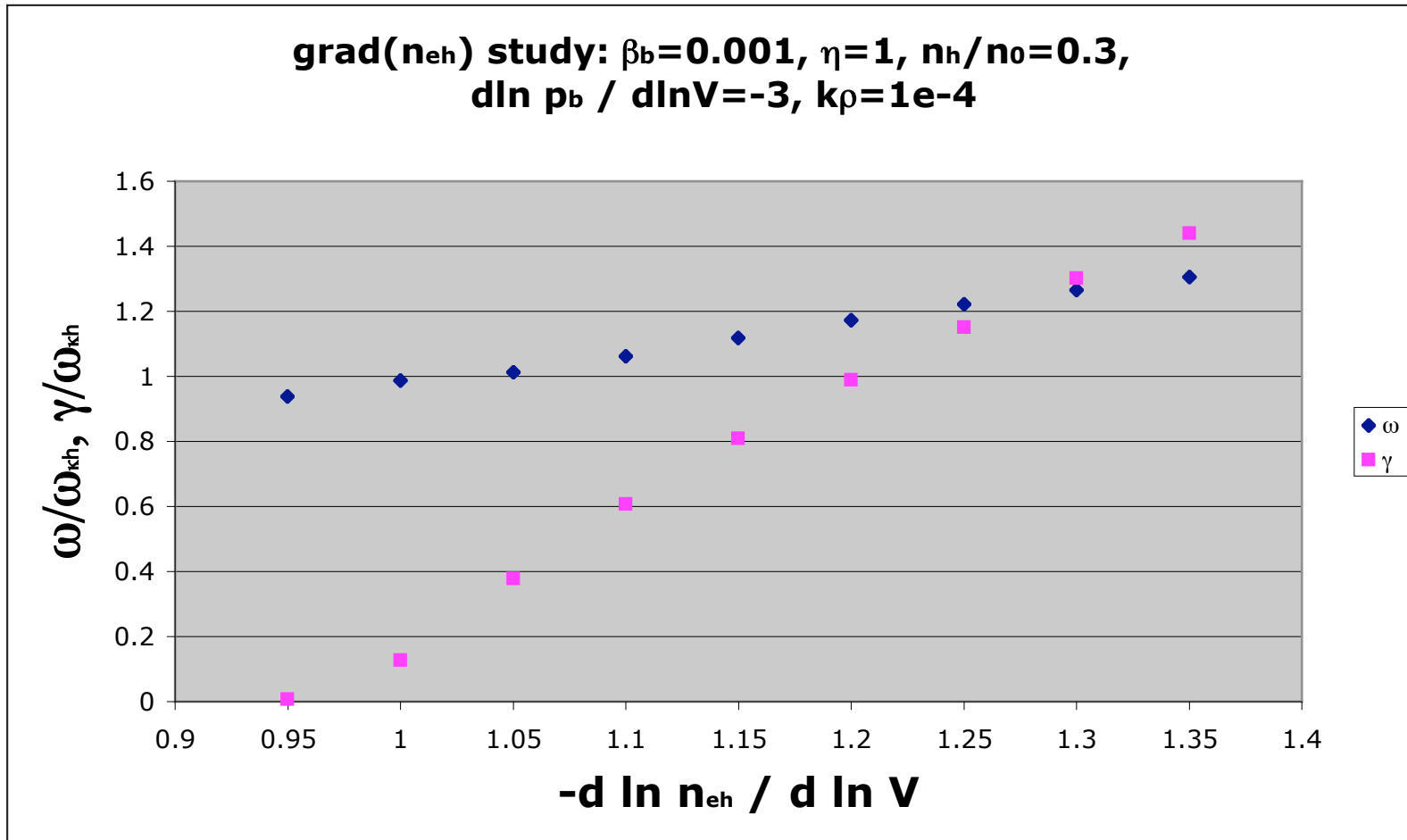$T_{eb}->0$ black

Beta affects will modify HEI

- β will modify the equilibrium (main effect) and introduce $\tilde{B}$.
- β-scan: fix background and n_{eh}/n_e. Increase T_{eh} and β_h.
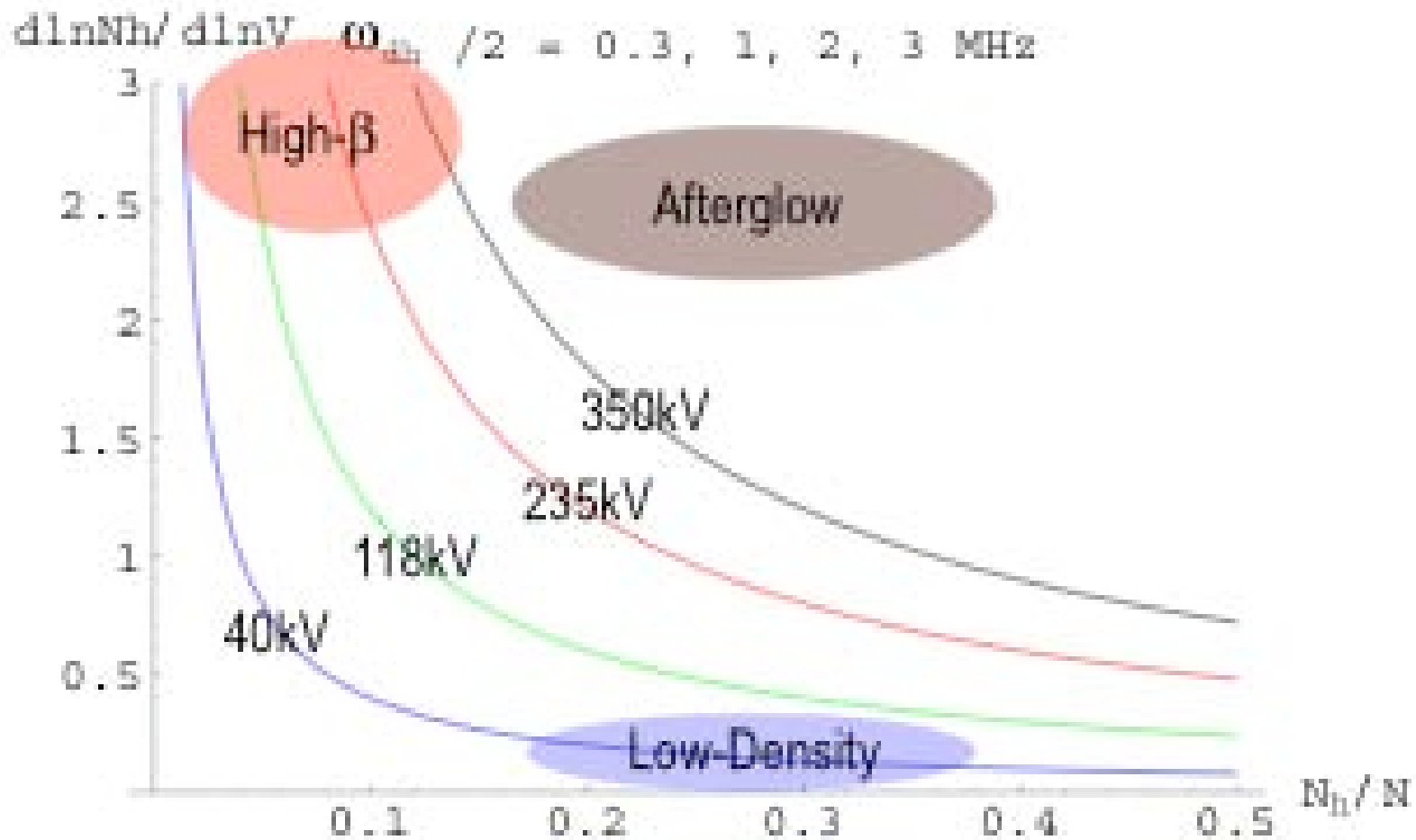- HEI growth rate asymptotes to higher value.

βh study: β_b=0.015, η=1, n_h/n_0=0.2, dln p_b / dlnV=-3, kρ=1e-4
HEI(black) and MHD (red)

HEI stable when $-d\ln n_{eh}/d\ln V < 1$

grad(n_{eh}) study: $\beta_b=0.001$, $\eta=1$, $n_h/n_0=0.3$, $d\ln p_b / d\ln V = -3$, $k\rho=1e\text{-}4$

- $-d\ln p_{eh}/d\ln V$ can exceed γ
- Density (not pressure) gradient determines HEI stability

LDX Operating Regimes

- LDX plasma exhibits 3 operating regimes: high density, high-β and after glow.

Non-Linear HEI simulation

Flux-Tube Integrated Dynamics
Gyrokinetic Electrons and Cold Ion Fluid Coupled through 2D Electric Fields

Electrons ($F \propto n_e V$)

$$\dot{\varphi} = \frac{\partial \mathcal{H}}{\partial \psi} = \mu \frac{c}{e} \frac{\partial B}{\partial \psi} - c \frac{\partial \Phi}{\partial \psi}$$

$$\dot{\psi} = -\frac{\partial \mathcal{H}}{\partial \varphi} = c \frac{\partial \Phi}{\partial \varphi}$$

Curvature

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial \varphi}(\dot{\varphi} F) + \frac{\partial}{\partial \psi}(\dot{\psi} F) = 0$$

Ions ($N \propto n_i V$)

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi}(N_i \| \nabla \varphi \cdot \mathbf{V} \|) + \frac{\partial}{\partial \psi}(N_i \| \nabla \psi \cdot \mathbf{V} \|) = 0$$

Gravity

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi}\left[cN_i \left(\omega_g(\psi) - \frac{\partial \Phi}{\partial \psi} - \left\| \frac{|\nabla \varphi|^2}{\omega_{ci} B} \right\| \frac{\partial^2 \Phi}{\partial \varphi \partial t} \right) \right]$$

$$+ \frac{\partial}{\partial \psi}\left[cN_i \left(\frac{\partial \Phi}{\partial \varphi} - \left\| \frac{|\nabla \psi|^2}{\omega_{ci} B} \right\| \frac{\partial^2 \Phi}{\partial \psi \partial t} \right) \right] = 0$$

$$\frac{\partial}{\partial \varphi}\left(h_\varphi \frac{\partial \Phi}{\partial \varphi} \right) + \frac{\partial}{\partial \psi}\left(h_\psi \frac{\partial \Phi}{\partial \psi} \right) = -4\pi e (N_i - N_e)$$

Electric Potential

Summary

- Hot electron pressure gradient can exceed the MHD limit. Consistent with experiments.

- Background plasma limited by MHD. Expected to exhibit convective flows when critical pressure gradient exceeded in background plasma.

- Non-linear simulations reproduce observed mode chirping.

- Exploration of ECH plasmas progressing in LDX

Website: www.psfc.mit.edu/ldx/

Flux-Tube Integrated Dynamics
Gyrokinetic Electrons and Cold Ion Fluid Coupled through 2D Electric Fields

Electrons ($F \propto n_e V$)

$$\dot{\varphi} = \frac{\partial \mathcal{H}}{\partial \psi} = \mu \frac{c}{e} \frac{\partial B}{\partial \psi} - c \frac{\partial \Phi}{\partial \psi}$$

$$\dot{\psi} = -\frac{\partial \mathcal{H}}{\partial \varphi} = c \frac{\partial \Phi}{\partial \varphi}$$

Curvature

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial \varphi}(\dot{\varphi} F) + \frac{\partial}{\partial \psi}(\dot{\psi} F) = 0$$

Ions ($N \propto n_i V$)

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi}(N_i \|\nabla \varphi \cdot \mathbf{V}\|) + \frac{\partial}{\partial \psi}(N_i \|\nabla \psi \cdot \mathbf{V}\|) = 0$$

Gravity

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi}\left[c N_i \left(\omega_g(\psi) - \frac{\partial \Phi}{\partial \psi} - \left\| \frac{|\nabla \varphi|^2}{\omega_{ci} B} \right\| \frac{\partial^2 \Phi}{\partial \varphi \partial t} \right) \right]$$

$$+ \frac{\partial}{\partial \psi}\left[c N_i \left(\frac{\partial \Phi}{\partial \varphi} - \left\| \frac{|\nabla \psi|^2}{\omega_{ci} B} \right\| \frac{\partial^2 \Phi}{\partial \psi \partial t} \right) \right] = 0$$
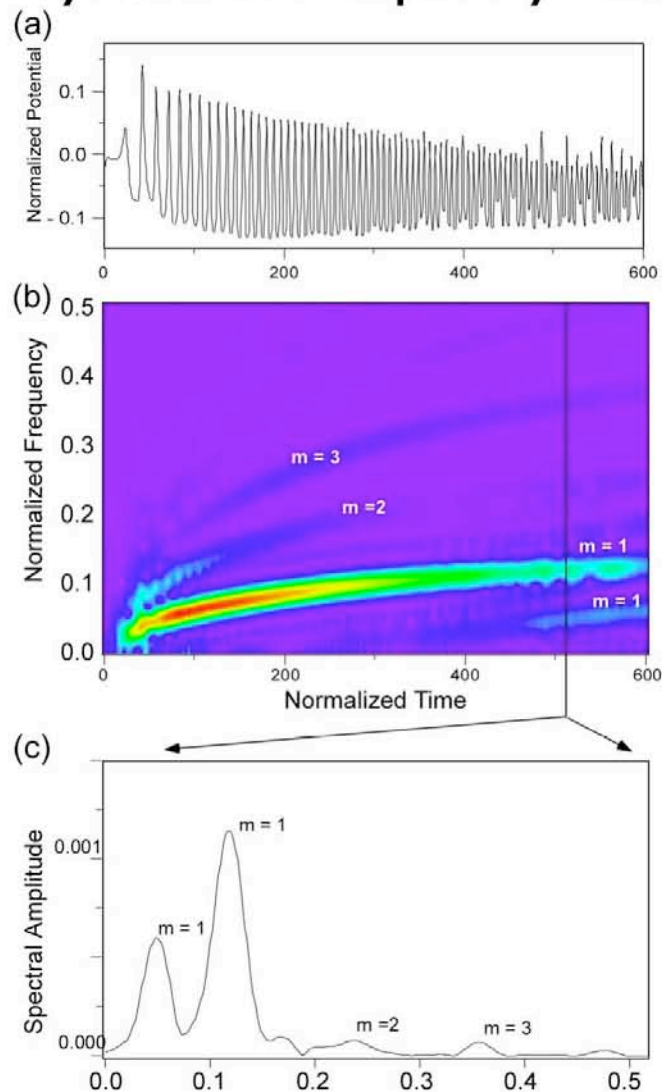
$$\frac{\partial}{\partial \varphi}\left(h_\varphi \frac{\partial \Phi}{\partial \varphi} \right) + \frac{\partial}{\partial \psi}\left(h_\psi \frac{\partial \Phi}{\partial \psi} \right) = -4\pi e (N_i - N_e)$$
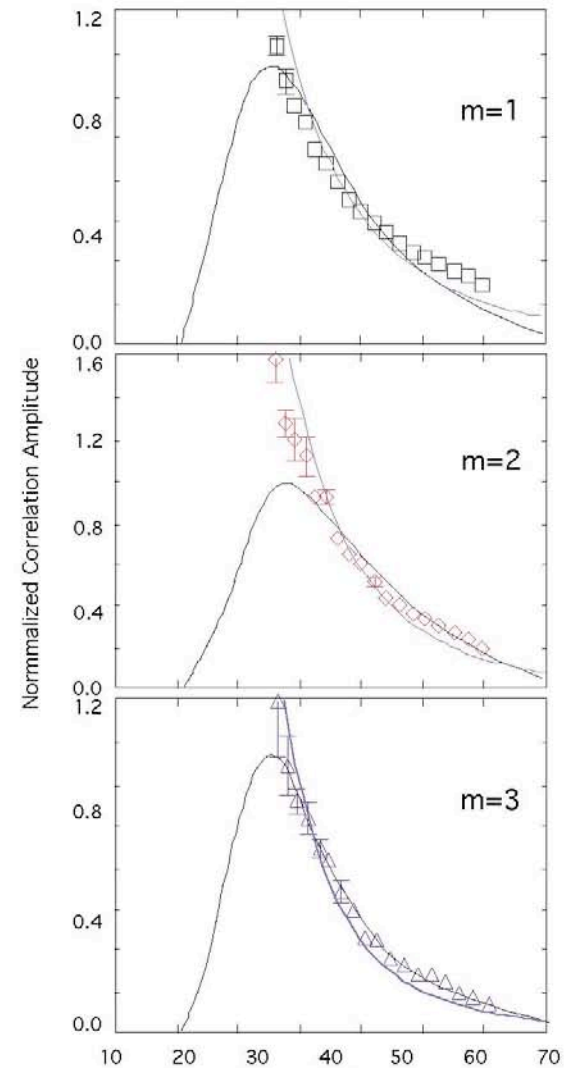
Electric Potential
(Constant along B-line & small dissipation)

Self-Consistent, Nonlinear, **Multi-Fluid**, **Field-Line Integrated,**
D Simulation Reproduces Dipole Interchange Dynamics and Mode Structu
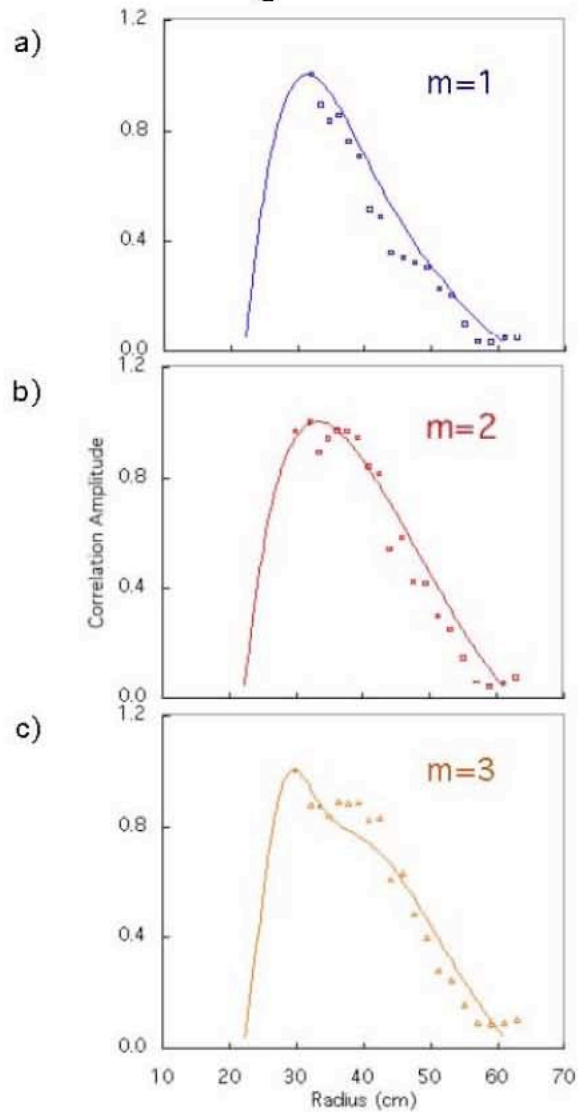
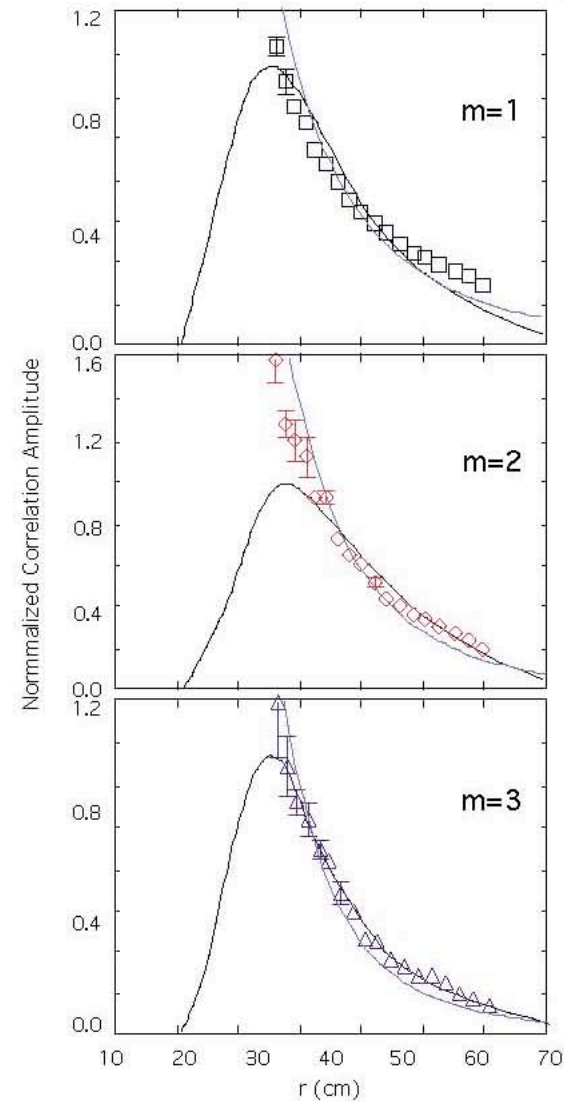## Dynamics: Frequency Rise

Structure: Broad, Multi-Mode

Dipole Interchange Modes have **Broad** Radial Structures
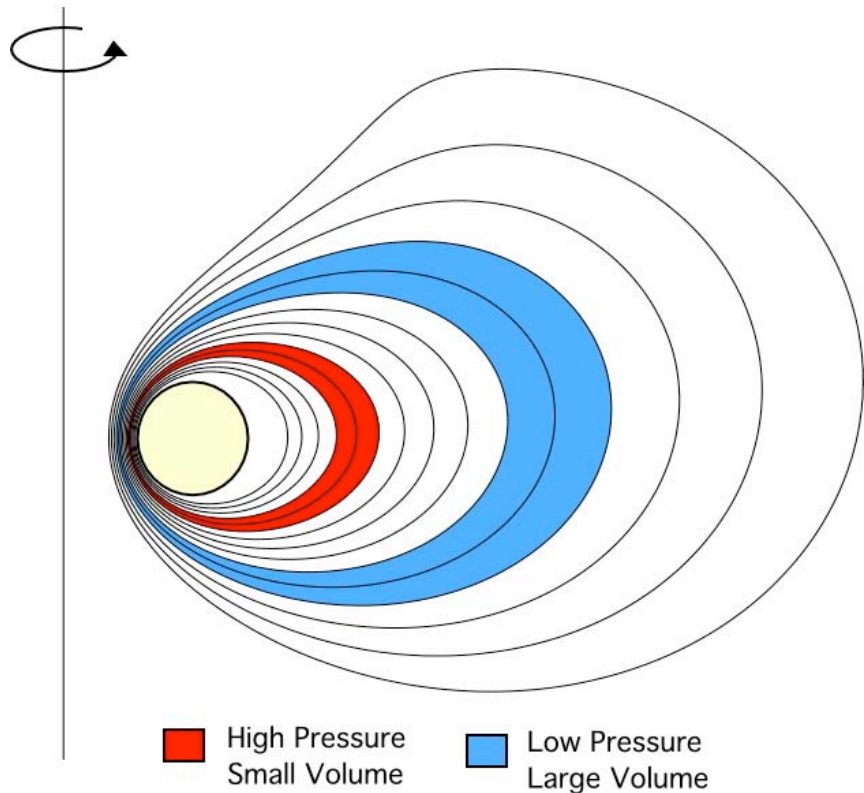
Centrifugal Interchange

Hot Electron Interchange

(2D Poisson's Equation: Computed mode structure shown with solid lines.)

Dipole Plasma Confinement

If $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$, then interchange does not change pressure profile.

For $\eta = \dfrac{d \ln T}{d \ln n} = \dfrac{2}{3}$, density and temperature profiles are also stationary.

- Toroidal confinement without toroidal field
  - Stabilized by plasma compressibility
  - No neoclassical effects
  - No TF or interlocking coils
  - p' constraint -> small plasma in large vac vessel
- Poloidal field provided by internal coil
  - Steady-state w/o current drive
  - $J_{||} = 0$ -> no kink instability drive

Floating Coil Produces Strong Dipole Field