Abstract

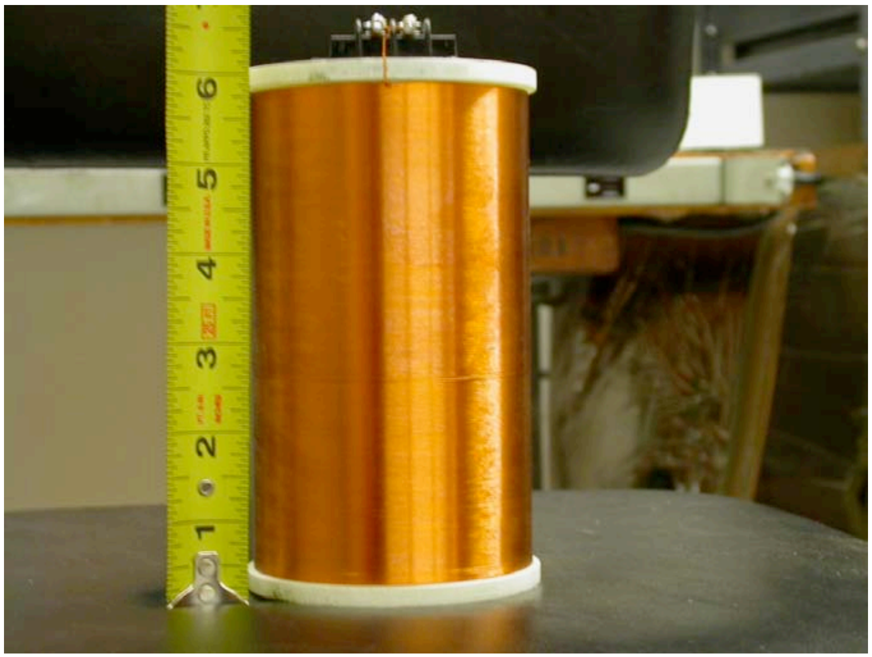
We report measurement of the equilibrium plasma current profiles in the Levitated Dipole Experiment (LDX) that exhibit a peak beta in excess of 10 percent. The beta of an LDX plasma is calculated by solving the Grad-Shafranov equation using the plasma current profile determined from magnetic measurements. The relevant magnetic sensors include nine pick-up coils normal to the vessel surface, nine coils parallel to the surface, and eight magnetic flux loops. Since the LDX dipole field is produced by a superconducting current ring, the dipole current decreases as the plasma current increases. Equilibrium profiles using different pressure models have been investigated. We find that the magnetic measurements primarily determine the plasma dipole moment, and additional constraints, including ECRH resonance zone locations and x-ray emission profiles, are needed to uniquely specify a pressure profile. The reconstruction results will be discussed along with the conditions that lead to the creation of high beta plasmas.

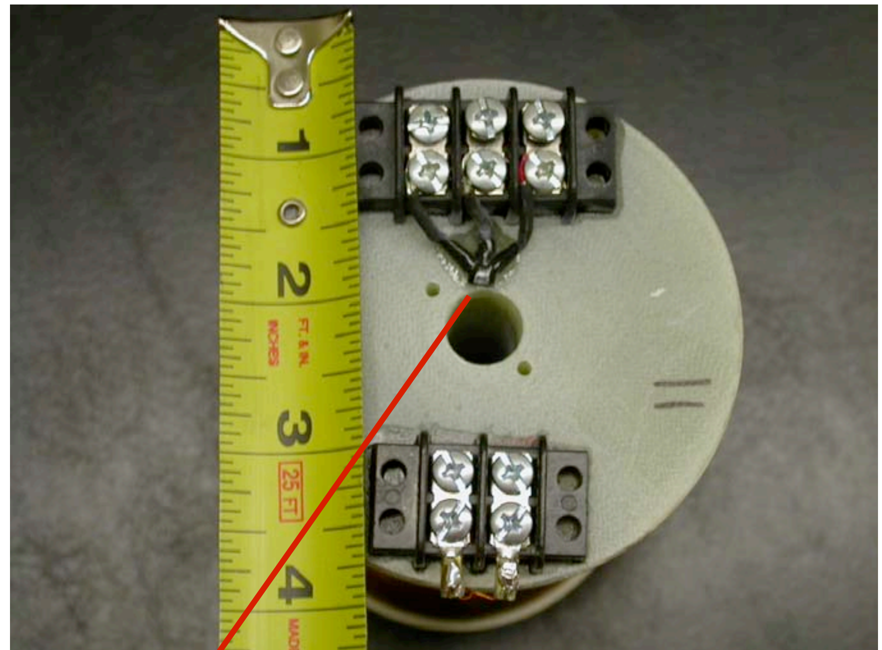
Outline

- Magnetic diagnostics basics
- Different pressure models
- Current filament code
- Reconstruction techniques and results
- Anisotropic pressure effects
- Summary / Future Work

Magnetic Diagnostics Basics

Sensors that measure magnetic field and flux.





Hall Probe

B_p-Coil Specs:

- NA ~ 5 m^2
- Sensitivity: 500 mV/G (connected to a 1 ms RC integrator)
- +/- 0.1 G estimated error

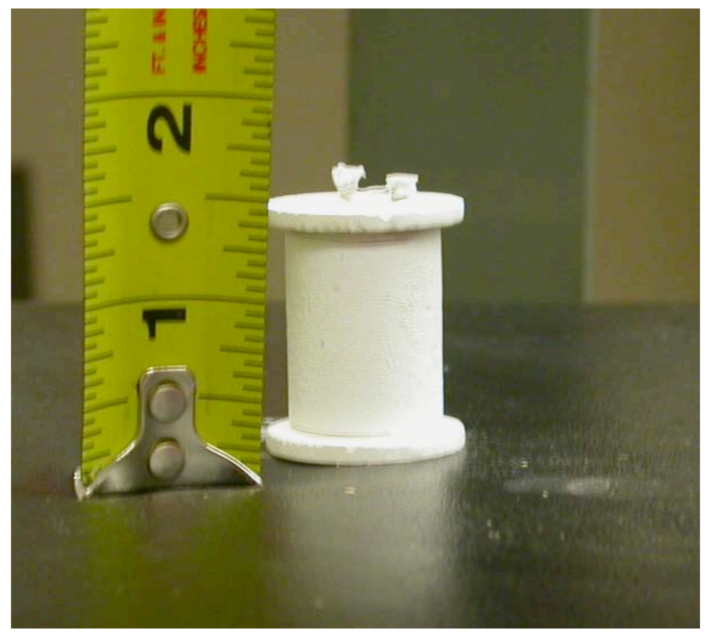
Hall Sensor Specs:

- Field Range: +/- 500 G
- Sensitivity: 5 mV/G

Flux Loop:

- Measures magnetic flux.
- Signal is integrated.
- +/- 0.1 mV·s estimated error

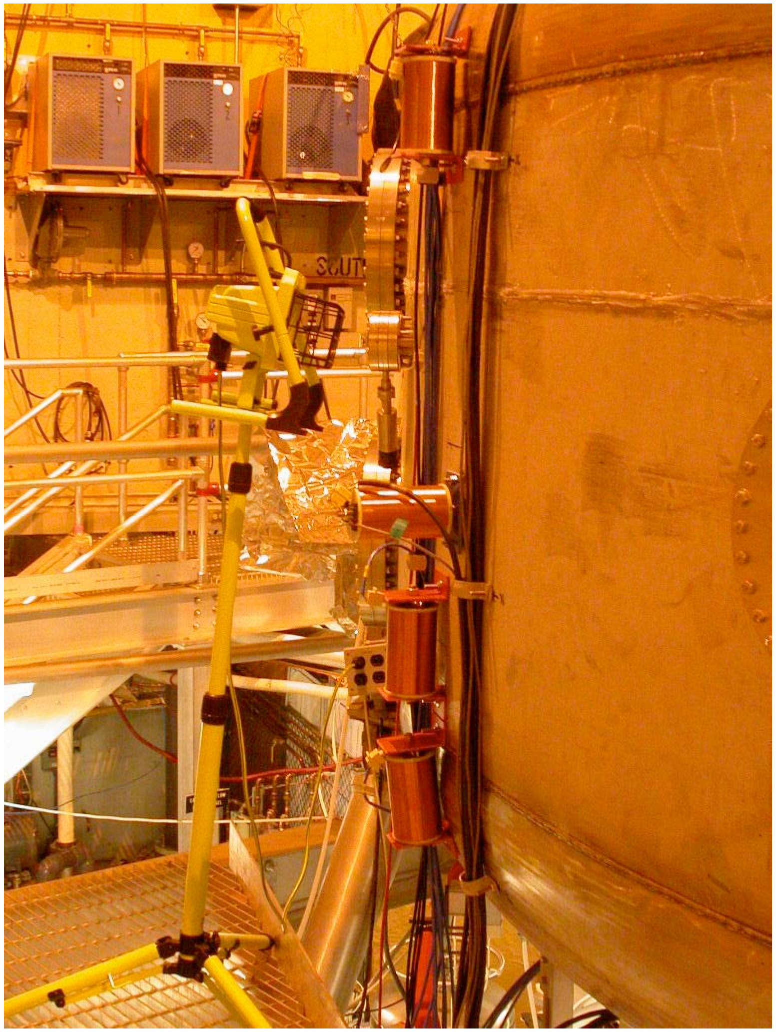
Mirnov Coil:



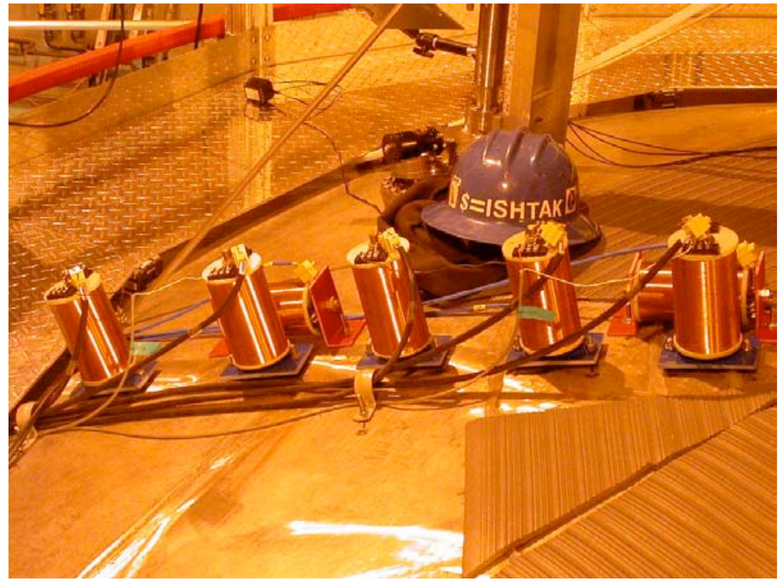
- Directly measures dB/dt to detect magnetic fluctuations.
- Must be placed inside the vessel to be able to measure fast activities.

NA ~ 0.06 m^2
L ~ 0.3 mH

Chamber Side



Chamber Top



Chamber Bottom

Magnetics Overview

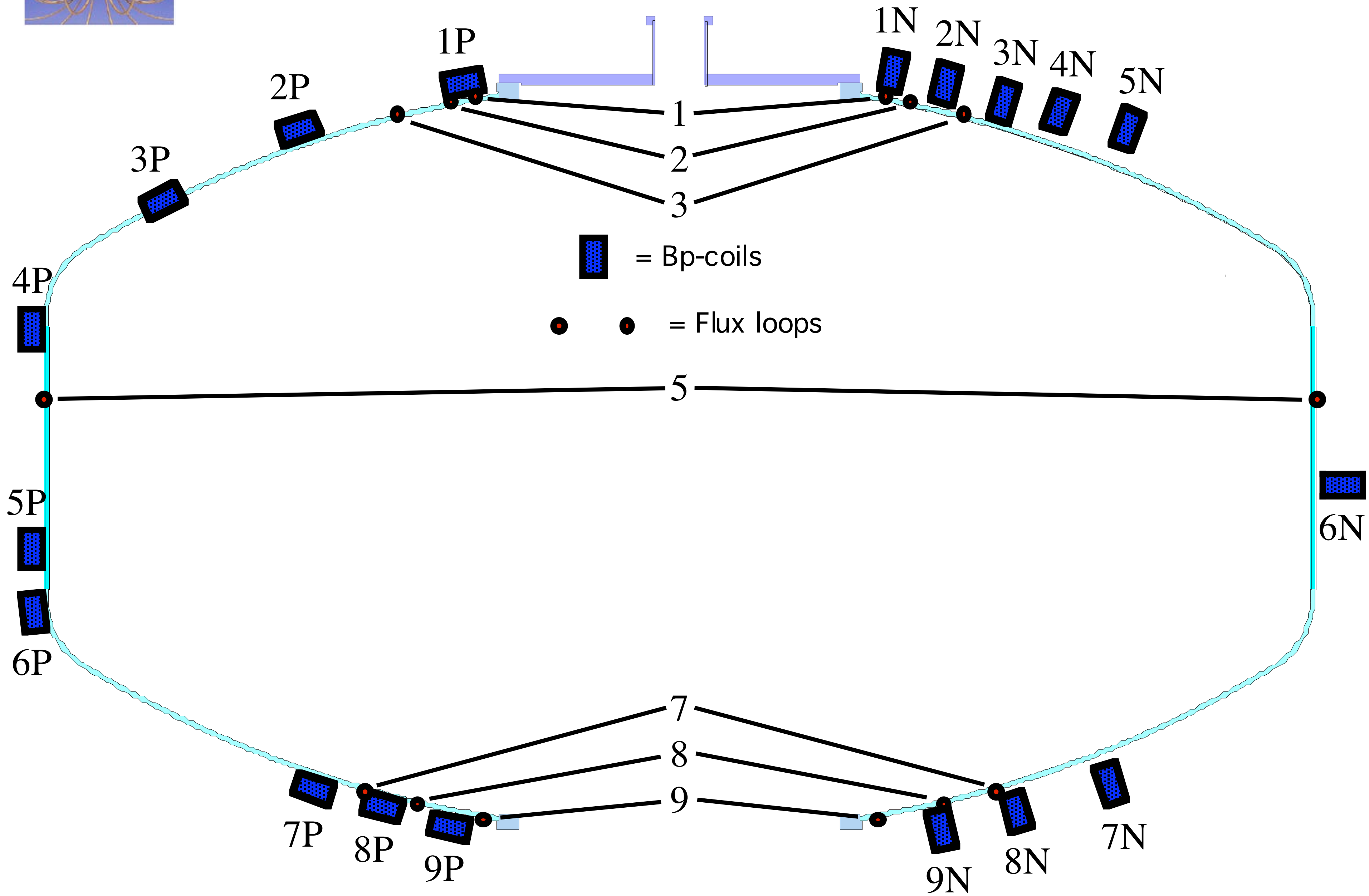
- 18 B_p-coils, half of which are oriented normal to the vacuum vessel and the other half oriented tangentially, to measure boundary diamagnetic field values.

- 8 flux loops to measure boundary diamagnetic flux values (total of 9 in the near future).

- 18 Hall probes, each mounted on a B_p-coil, to supplement the coil measurements.

- 2 toroidally separated Mirnov coils to measure fast plasma fluctuations (total of 8 in the near future).

Sensor Postions



Importance of Magnetic Diagnostics

We can deduce the following plasma parameters through magnetic measurements:

- Current and pressure profiles
- Plasma shape and position
- Average and peak beta

Substantial analyses must be performed on the magnetic data to actually obtain the above mentioned parameters.

Reconstruction Basics

Magnetic field - Pressure relation: Grad-Shafranov equation

$$\boxed{\Delta^*\psi = -\mu_0 R^2 \frac{dP}{d\psi}}$$

$$J_\phi = \frac{-1}{\mu_0 R}\Delta^*\psi \qquad \Delta^* \equiv R^2\nabla\cdot\left(\frac{\nabla}{R^2}\right)$$

$$P = P(\psi)$$

The functional form of P(ψ) is usually unknown, so we typically construct a functional model with free parameters and use the magnetic data to constrain them.

Candidate Pressure Models

$$P(\psi;\psi_{peak},P_{edge},g) = \begin{cases} P_{edge}[\frac{V_{edge}}{V(\psi)}]^g & \text{for } \psi > \psi_{peak} \\ P_{edge}[\frac{V_{edge}}{V(\psi)}]^g \sin^2[\frac{\pi}{2}(\frac{\psi}{\psi_{peak}})^2] & \text{for } \psi < \psi_{peak} \end{cases}$$

$$V \equiv \oint \frac{dl}{B}$$

"DipoleEq profile"

$$P(\psi;a,b,g) = \frac{a}{V^g}(\psi_{max}-\psi)^b(\psi-\psi_{min})^c$$

"No edge pressure profile"

$$P(\psi;\psi_{peak},P_{peak},g) = P_{peak}(\frac{\psi-\psi_{f-coil}}{\psi_{peak}-\psi_{f-coil}})^\alpha(\frac{\psi}{\psi_{peak}})^{4g}$$

$$\alpha \equiv 4g(|\frac{\psi_{f-coil}}{\psi_{peak}}|-1)$$

"Smooth adiabatic profile"

Relevant Parameters to be Obtained

When the Grad-Shafranov equation is numerically solved with a proper minimization scheme to solve for the free parameters, we will have computed ψ(R, Z) and P(Ψ).

Then:

- $\psi(R, Z)$ will directly give the plasma shape and position.
- Toroidal current, $J_\phi = R\dfrac{dP}{d\psi}$
- Peak beta, $\beta_{peak} = \dfrac{2\mu_0 P}{B_\theta^2}|_{peak}$
- Average beta, $\beta_{average} = \dfrac{2\mu_0 \langle P \rangle}{\langle B_\theta^2 \rangle}$

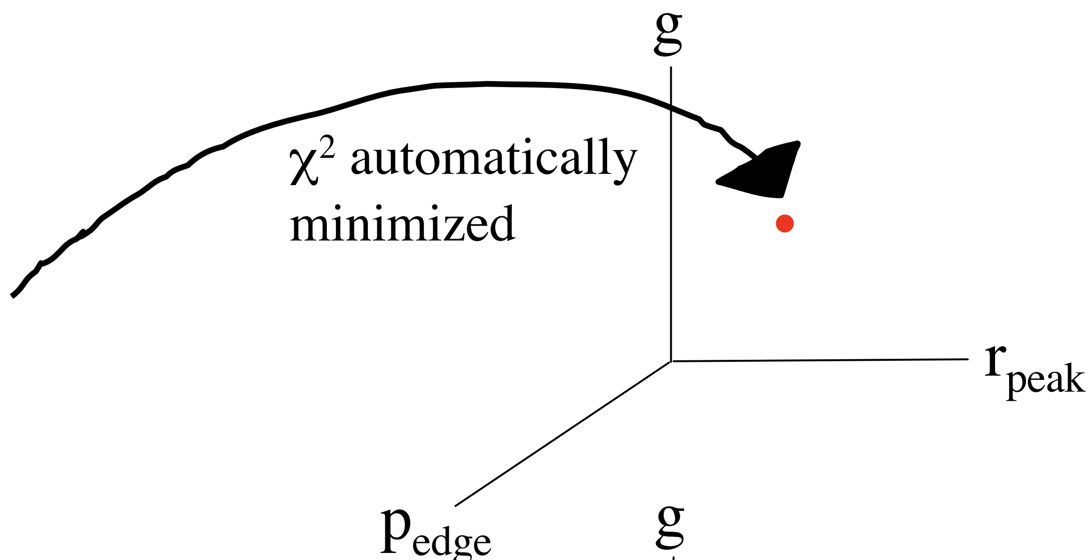
Hence, the aforementioned parameters can be easily calculated from ψ(R, Z) and P(Ψ).

Full Reconstruction Method

The code we have today does not have the fitting capability, hence a “poor man’s” fitting routine is performed.

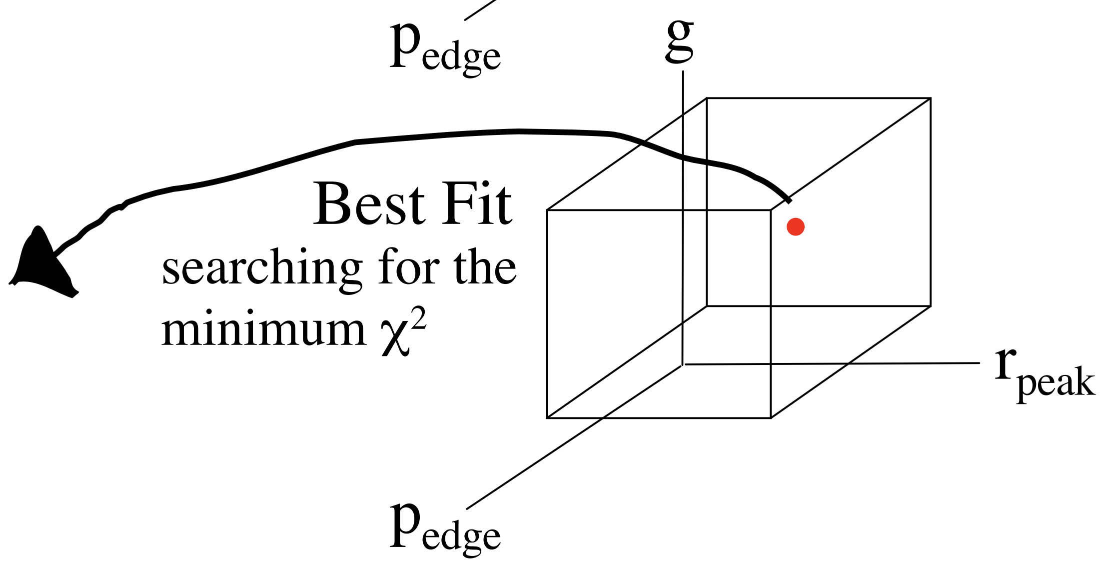
Ideal code operation:

$$[B_1, B_2, \ldots, B_{18}]$$
$$[\Psi_1, \Psi_2, \ldots, \Psi_9]$$



Current code operation:

$$[B_1, B_2, \ldots, B_{18}]$$
$$[\Psi_1, \Psi_2, \ldots, \Psi_9]$$



Pseudo-Reconstruction using the Vacuum Field

- Pick a pressure model and make an initial guess on the free parameters.
- Use the vacuum flux ψ_{vac} in the relation $J_\phi = R*dP/d\psi$ to get the corresponding current.
- Calculate χ^2.
- Tweak the free parameters until the best fit is found.

Advantage: - Can forgo the G-S equation and save on computing time.

Disadvantage: - Accurate only for low plasma currents (< 20 kA).

DFIT: The Current Filament Code

- Essentially an MFIT in the dipole geometry.
- A two current filament code.
- Capable of holding the F-coil flux constant while changing the current filaments' magnitude and location.
- Inputs are all magnetic measurements.
- Outputs include total plasma current, current centroid, change in F-coil current due to the presence of plasma current, and the fit parameter χ^2.
- The code runs in real time during a shot.

Plasma Effect on the F-coil Current

- **Because the flux contained by F-coil must be conserved, the plasma co-current acts to reduce the F-coil current.**

$$L_f I_f = L_f(I_f + \Delta I_f) + M_{fp} I_p$$

$$\therefore \Delta I_f = -\frac{M_{fp}}{L_f} I_p$$

$$M_{fp} I_p \equiv \sum_{i=1}^{N} \int\int M_i(x,z) J_p(x,z)\, dx dz$$

Typically, $L_f \sim 5 M_{fp}$

$I_p \sim 2500A$

$I_f \sim 1MA$

$$\frac{\Delta I_f}{I_f} \sim -\frac{500A}{1MA} \sim -0.05\%$$

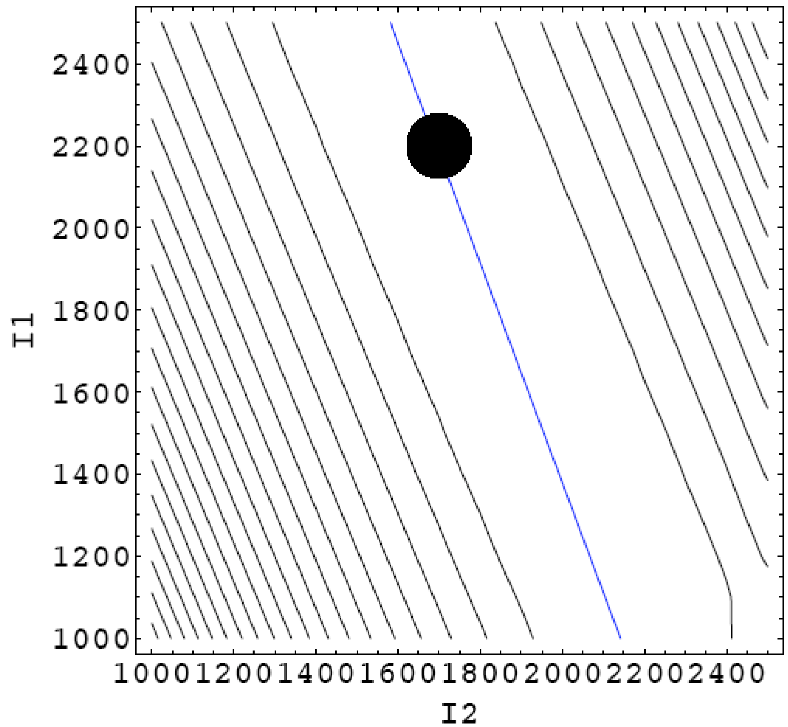
Magnetic diagnostics detect the sum of the changes in the field caused by the plasma diamagnetic current and the decrease in the F-coil current!

Sensitivity of χ^2 to the Plasma Magnetic Moments

- Reconstruction results show that the LDX magnetic diagnostics are sensitive only to the plasma dipole moment:

Example (shot 50318014 @ t = 3 s):

χ^2 Contours for the 2 Filament DFIT Model at Fixed Filament Locations



The blue line represents a contour of constant dipole moment. This line is essentially identical to the line of minimum χ^2 signifying that the magnetic measurements cannot decipher between different current profiles with the same dipole moment.

Sensitivity of χ^2 to the Plasma Magnetic Moments (continued)

There are no currents where the magnetic sensors are located:

$B = \nabla \eta$, where η is the magnetic scalar potential.

We can expand η as a sum of spherical harmonics:

$$\eta = \sum_l A_l r^{-(l+1)} Y_{l0}(\theta)$$

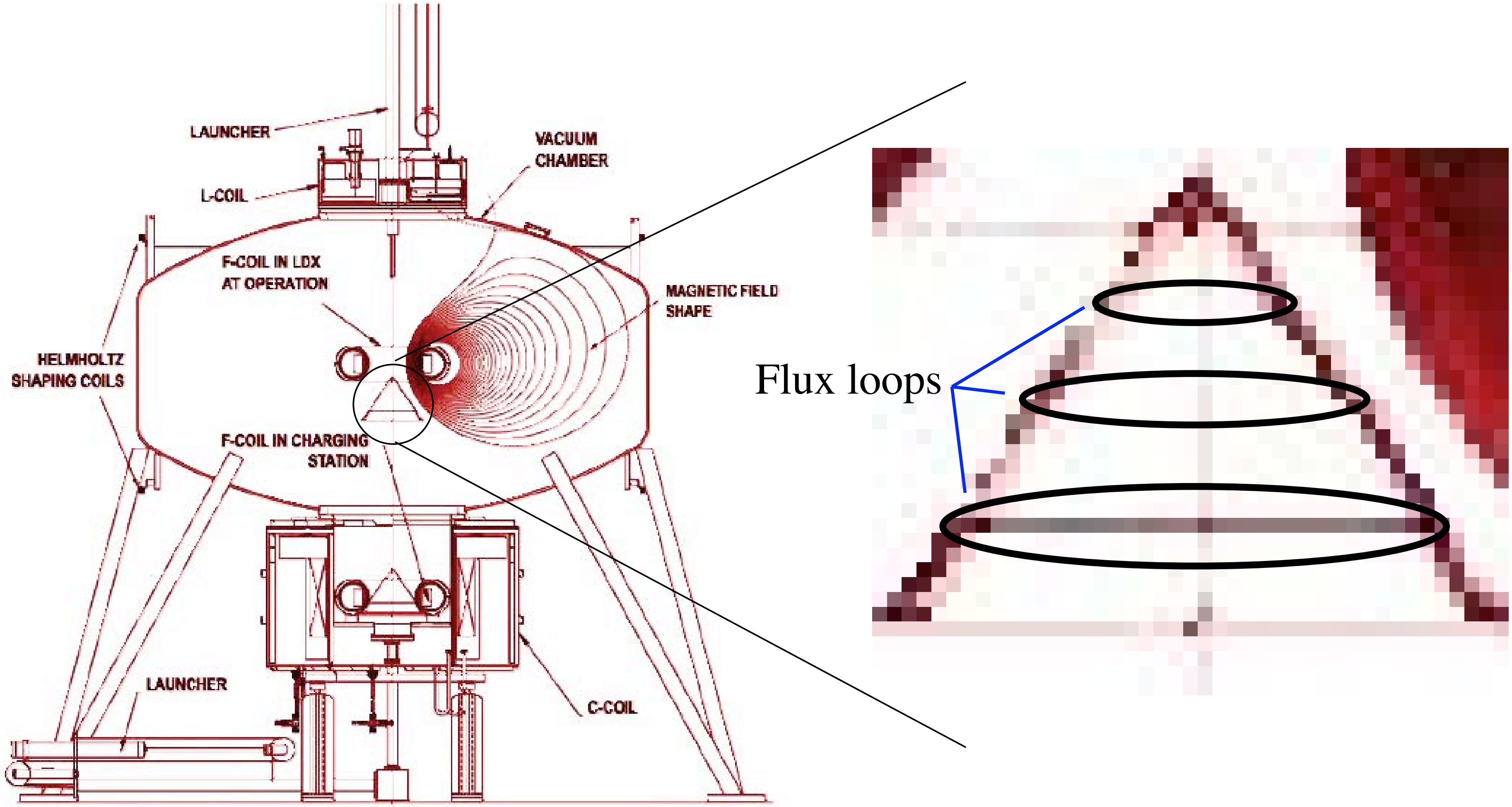
$A_0 = 0$ since no monopole moment.

A_1 (dipole) is the dominant term for large r where the magnetic sensors are. Furthermore, the quadrupole moment is dominated by the induced change in the F-coil current and hence very difficult to measure. Higher order moments are even harder to measure.

Remedy: Put magnetic sensors closer to the plasma and F-coil!

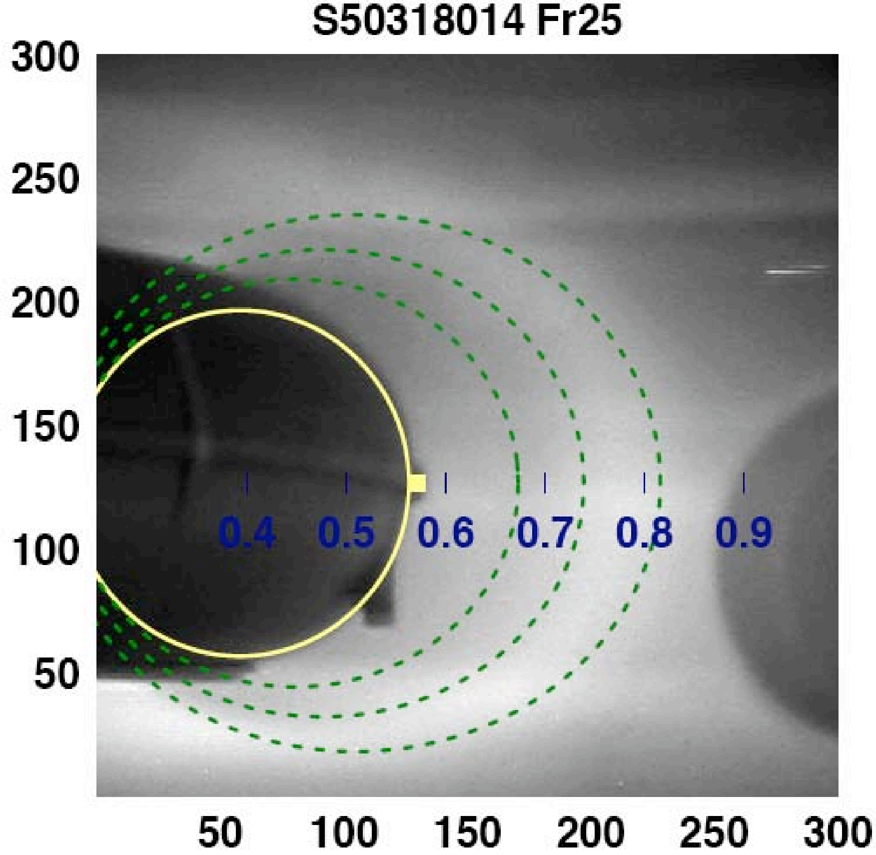
Sensitivity of χ^2 to the Plasma Magnetic Moments (continued)

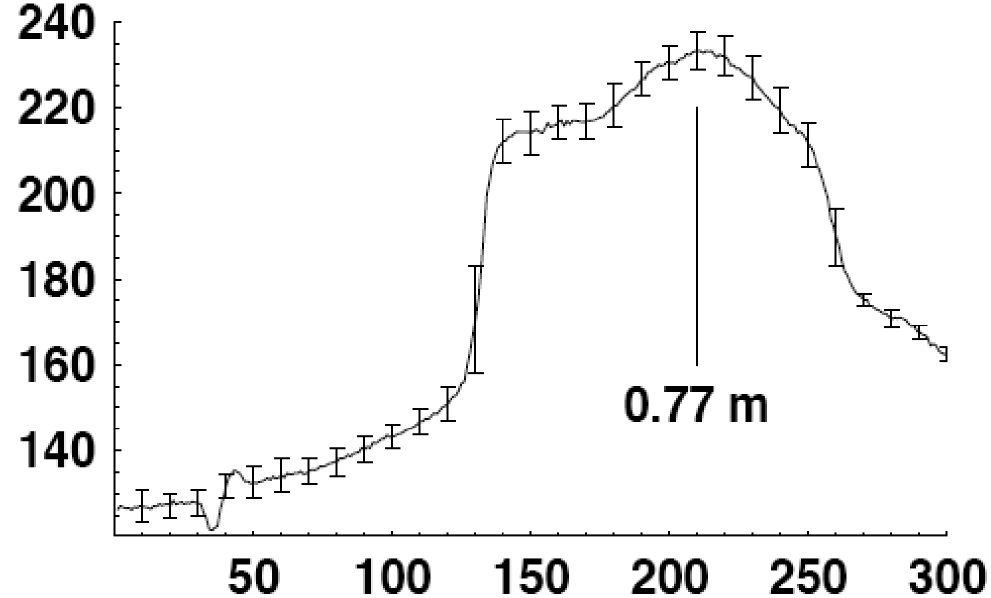
A possible location to put more flux loops near the plasma:



Sensitivity of χ^2 to the Plasma Magnetic Moments (continued)

X-ray measurements can give you an additional profile information assuming that the peak pressure location coincides with the emissivity peak:





Sensitivity of χ^2 to the Input Parameters

(shot 40917019 @ 3.22 sec)

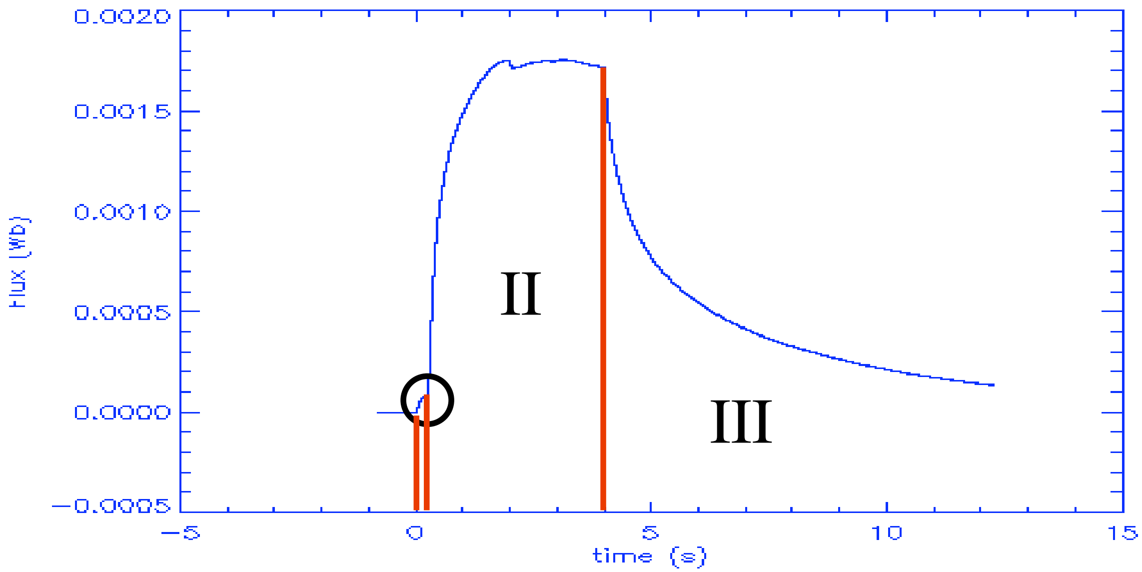
$\chi^2_{min} \approx 25$

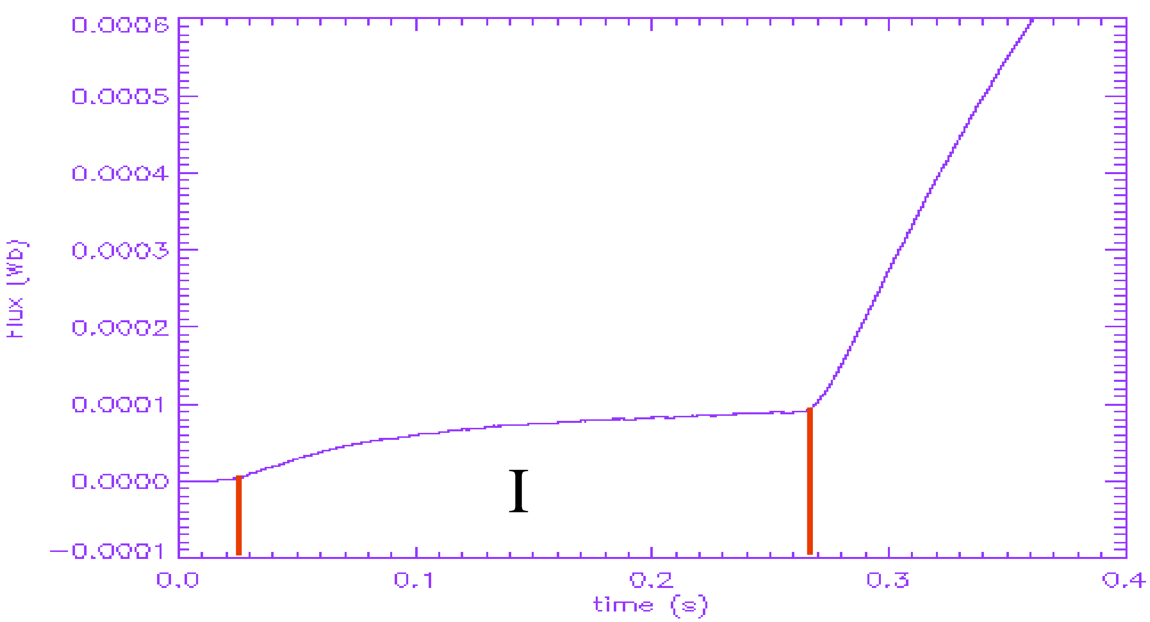
δp_{edge} = +/- 0.001 Pa → $\delta\chi^2$ = +17 / +20

δr_{peak} = +/- 0.01 m → $\delta\chi^2$ = +2 / +1

$\delta f_{crit} = (3/5)\delta g$ = +/- 0.01 → $\delta\chi^2$ = +8 / +9

A Typical Plasma Shot: The 3 Regimes



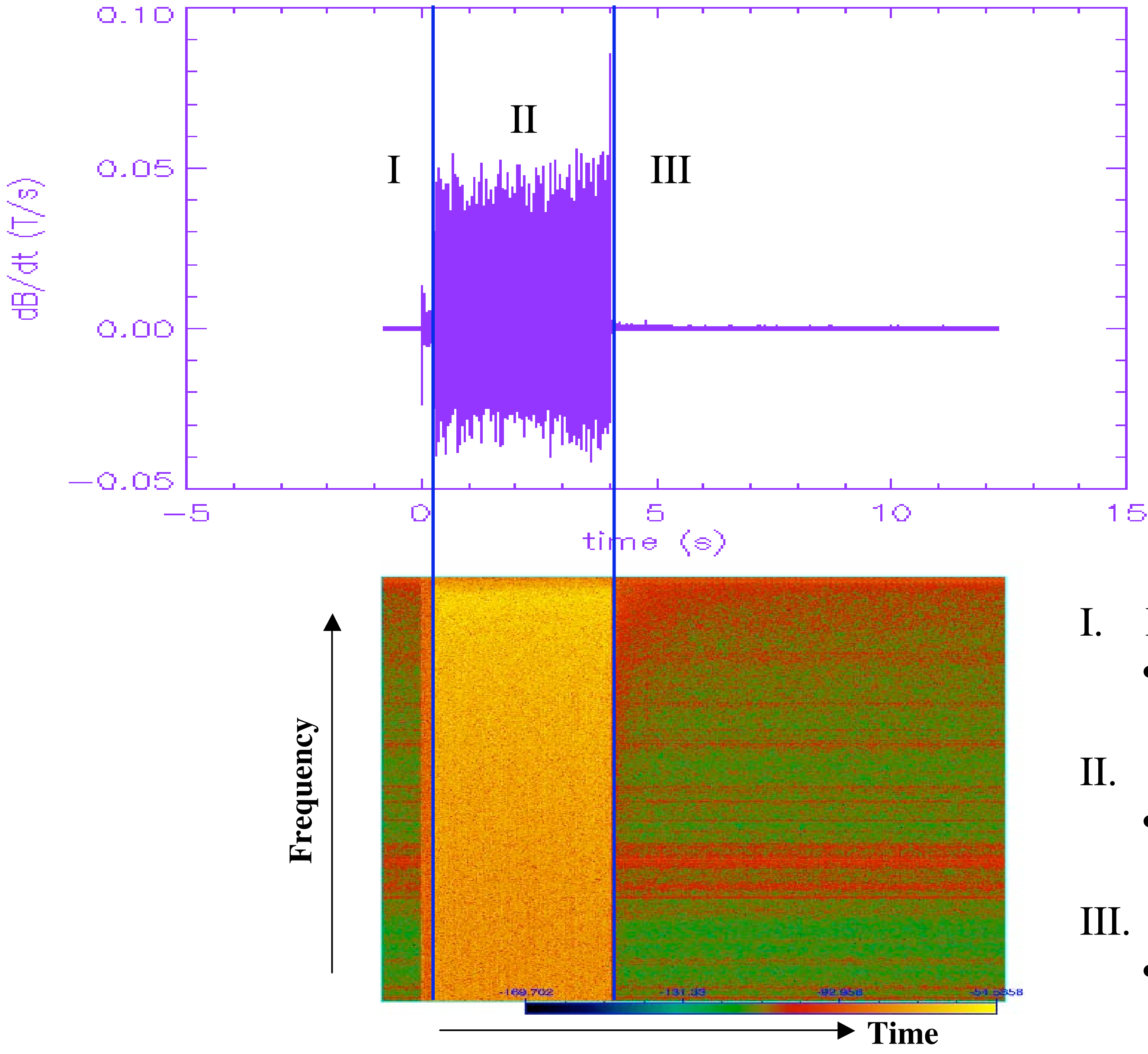


I. Low-density region
- Low bulk density
- Very slow rise in diamagnetism

II. High-beta region
- Higher bulk density
- Rapid rise in diamagnetism

III. After-glow region
- Bulk plasma gone
- Slow decay in diamagnetism

A Typical Plasma Shot: The 3 Regimes



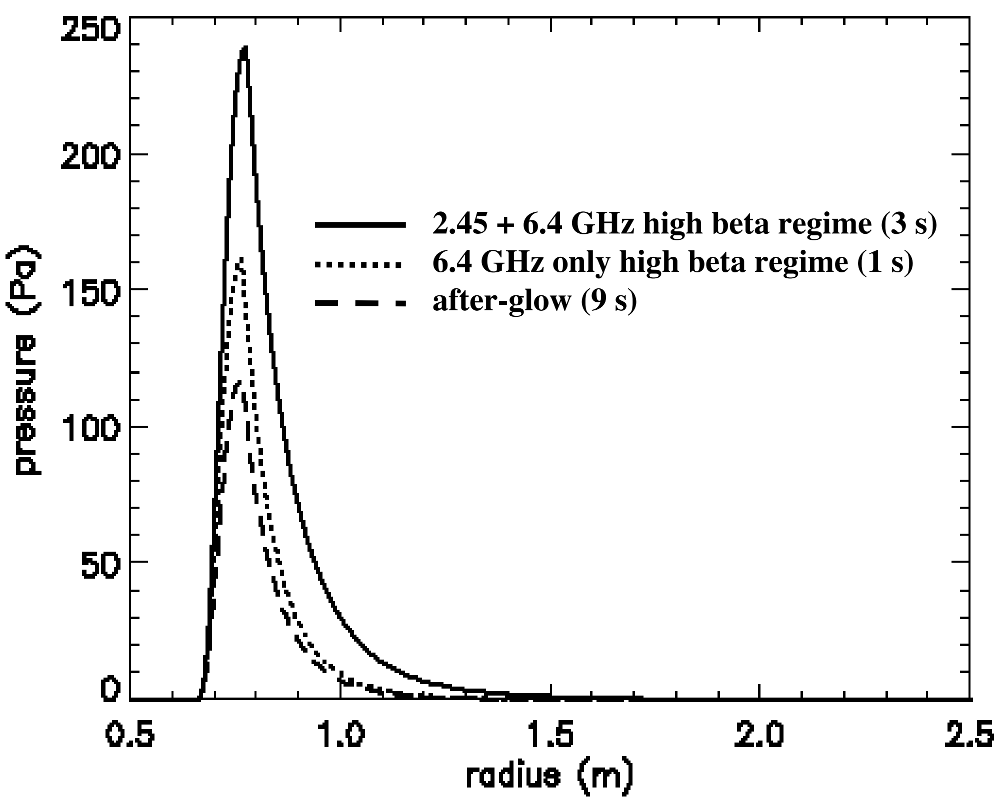
I. Low-density region
   - Low Mirnov activity

II. High-beta region
   - High Mirnov activity

III. After-glow region
   - No Mirnov activity

High Beta Shot

Shot 50318014 Full Reconstruction Results at Three Different Times:



	6.4 GHz only high beta	2.45 + 6.4 GHz high beta	After-glow
Peak beta	5.8 %	10.0 %	4.1 %
Volume averaged beta	0.21 %	1.4 %	0.17 %
Total stored energy	73 J	161 J	55 J
Plasma volume	28.5 m^3	28.3 m^3	28.5 m^3
Plasma current	1247 A	2808 A	934 A

Model used:

$$P(\psi;\psi_{peak},P_{edge},g) = \begin{cases} P_{edge}[\frac{V_{edge}}{V(\psi)}]^g & \text{for } \psi > \psi_{peak} \\ P_{edge}[\frac{V_{edge}}{V(\psi)}]^g \sin^2[\frac{\pi}{2}(\frac{\psi}{\psi_{peak}})^2] & \text{for } \psi < \psi_{peak} \end{cases}$$

High Beta Shot

Shot 50318014 “Vacuum” Reconstruction Results at t = 3s:

Parameters \ Model Type	**DipoleEq**	**No Edge Pressure**	**Smooth Adiabatic**
Peak pressure (Pa)	310	647	162
Peak beta (%)	11	9.5	8.4
Volume averaged beta (%)	1.6	1.2	1.3
Total stored energy (J)	290	236	247
Plasma current (kA)	3.3	3.0	3.1
Current centroid (m)	0.93	0.97	0.97
Change in F-coil current (kA)	-0.87	-0.76	-0.77

Effect of Anisotropic Pressure on Reconstruction Results

$$\underline{\nabla} P = \underline{J} \times \underline{B} \longrightarrow \underline{\nabla} \cdot \underline{\underline{P}} = \underline{J} \times \underline{B}$$

scalar pressure — pressure tensor

<u>Perpendicular force balance</u>:

$$\underline{J} = \frac{\underline{B} \times \underline{\nabla} \cdot \underline{\underline{P}}}{B^2} = \frac{\underline{B} \times \underline{\nabla} P_\perp}{B^2} + \frac{\underline{B} \times \underline{\kappa}}{B^2}(P_\parallel - P_\perp)$$

$$J_\phi \approx R[\frac{\partial P_\perp}{\partial \psi} + (P_\parallel - P_\perp)\frac{\partial \ln B}{\partial \psi}]$$ (Vacuum field approximation)

$P_\perp > P_\parallel$ and $\frac{\partial \ln B}{\partial \psi} > 0$ outside the pressure peak.

Consquence: The total pressure is greater than that in the isotropic case for the same level of current. Hence, an anisotropic model predicts a larger β for a given measured current.

Summary/ Future Work

- The magnetic diagnostics have successfully been recording data from LDX plasmas.
- Different reconstruction techniques have been employed using different pressure models.
- Reconstruction results show that magnetic diagnostics alone are sensitive only to the plasma dipole moment.
- A current filament code, DFIT, has been developed to allow for a fast reconstruction of the current profile.
- A typical plasma shot consists of three different regimes that can clearly be seen on the magnetic sensors.
- An anisotropic pressure model predicts a higher beta for a given current.