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Drift Frequency Interchange Modes in a Dipole Confined Plasma at Varying Collisionality

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J. Kesner, J.E. Hastie

Plasma Science and Fusion Center, MIT

Cambridge, Ma. 02139

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Abstract

Gross plasma stability can derive from plasma compressibility in the bad curvature regions in closed field line systems such as in a dipole field. In this situation MHD theory predicts that the maximum pressure gradient that is stable is proportional to γ, the ratio of specific heats.

We have examined low β electrostatic modes using a kinetic approach in various collisionality regimes including the collisional regime [1,2], the collisionless ion regime expected in the LDX experiment and collisionless ion and electron [3], which might be expected in a reactor. We show that near marginal stability there is a coupling between the MHD-like mode and a low frequency "entropy" mode. The maximum sustainable pressure gradient was found to be dependent on the ratio of the temperature and density gradients ($\eta \equiv (n/T)(\nabla T/\nabla n)$) as well as on the curvature drift frequency. For $\eta = 2/3$ the MHD stability condition is reproduced. When $\eta < 2/3$ the mode changes character and the stability criterion becomes more stringent in all collisionality regimes.

Kesner, Phys Plasmas **7** (2000) 3837.

Simakov, Catto, Hastie, Phys Plas **8**, 4414 (2001).

Kesner, Phys Plasmas **5** (1998) 3675.

Kinetic Analysis of low-β Plasma

- Ideal MHD
  - Assumes adiabatic eq-of-state with $\gamma = 5/3$.
  - Ion FLR and $\eta_i \equiv (n_i \nabla T_i)/(T_i \nabla n_i)$ does not enter single fluid equations.

MHD $\rightarrow \frac{1}{p}\frac{dp}{d\psi} \leq 2\gamma\langle\kappa_\psi\rangle$ or $\hat{\omega}_{*p} \leq \gamma\hat{\omega}_d^{mhd}$

- There are several interesting orderings:
  - Ideal MHD (short mean free path, collisional)

    $\Omega_c > \nu > \omega_b > \omega_* \sim \omega_d \sim \omega$
  - Long mfp collisional

    $\Omega_c > \omega_b > \nu > \omega_* \sim \omega_d \sim \omega$
  - Semi-collisional (expected in LDX)

    $\Omega_{ce} > \omega_{be} > \nu_e > \omega_{*e} \sim \omega_{de} \sim \omega$

    $\Omega_{ci} > \omega_{bi} > \omega_{*i} \sim \omega_{di} \sim \omega > \nu_i$
  - Collisionless (expected in dipole reactor)

    $\Omega_c > \omega_b > \omega_* \sim \omega_d \sim \omega > \nu$

Kinetic Analysis

- From DKE obtain $\tilde{f} = q\phi F_{0\epsilon} + J_0(k_\perp \rho)h$.

  with the non-adiabatic response, h, determined from:

  $$\left(\omega - \omega_d + iv_{||}\vec{b}\cdot\nabla'\right) h = -(\omega - \omega_*)q\phi F_{0\epsilon} J_0(k_\perp\rho) + iC(h)$$

  Assuming high bounce frequency the non-adiabatic response $h = h_0$ satisfies

  $$(\omega - \overline{\omega}_d)\, h_0 = -\,(\omega - \omega_*)\, q\overline{\phi J_0} F_{0\epsilon} + i\overline{C}(h_0) \quad (1)$$

  with $\omega_* = \frac{\vec{b}\times\vec{k}_\perp\cdot\nabla' F_0}{m\Omega_c F_{0\epsilon}}$

  $$\omega_d = \vec{k}_\perp \cdot \vec{b} \times \frac{(v_{||}^2 \vec{b}\cdot\nabla\vec{b} + \mu\nabla B)}{\Omega_c},$$

  $$\overline{\phi} = \left(\oint \frac{\phi(l)\,dl}{\sqrt{1-\lambda B}}\right) / \left(\oint \frac{dl}{\sqrt{1-\lambda B}}\right) \text{ and } \lambda = \epsilon/\mu.$$

- Dispersion relation: Solve for h_0, integrate over velocity space, apply quasi-neutrality.

Long mean-free-path Collisional Regime (Entropy mode)

- For $\nu_i, \nu_e \gg \omega, \omega_*, \omega_d$ obtain $\overline{C}(h_0) \approx 0$. Therefore

  $$h_0 = \delta n \left(\frac{m/2\pi}{T+\delta T}\right)^{3/2} e^{-\epsilon/(T+\delta T)} \approx \left[\frac{\delta n}{n_0} + \frac{\delta T}{T}\left(\frac{\epsilon}{T} - \frac{3}{2}\right)\right] F_0$$

- Take the flux tube and velocity space average and assume the collision operator conserves particles and energy:

  $$\int dl/B \int d^3v \overline{C}(h) = \int dl/B \int d^3v(\epsilon - 3/2)\overline{C}(h) = 0.$$

- We can now integrate Eq. [1] to solve for δn and δT in terms of "fluid" frequencies,

  $$\hat{\omega}_{*j} = \frac{T\vec{k}_\perp \times \vec{b}\cdot\nabla n_0}{n_j m\Omega}$$

  and $\hat{\omega}_d = \frac{cT(Rk_\perp)}{qV} \int \frac{dl}{B^2 R}(\kappa + \nabla B/B)$.

  noting $\omega_{*pi}/\langle\omega_{di}\rangle \equiv \hat{\omega}_{*i}(1+\eta_i)/\langle\omega_{di}\rangle = -d\ln p/d\ln \nu$

  and $\overline{b} = \langle k_\perp^2 T_i/M_i\Omega_i^2\rangle$

- For $k_\perp \rho_i \sim 0$ obtain at marginal stability [Kesner, Phys Plasmas **7** (2000) 3837]

$$d = \frac{5}{7}\frac{1+\eta}{1-\frac{3}{7}\eta} \tag{2}$$

- Gyro-relaxation corrections: Simakov, Catto, Hastie Phys Plasmas **8**, 4414 (2001)
  - $h_1 \sim O(\omega_*/\nu_{ii}) \rightarrow$ introduce "gyro-relaxation" corrections.
  - Proved that mode is flute-like.

Collisionless Ions: Collisional Electrons (Semi-Collisional) Regime

Likely LDX regime

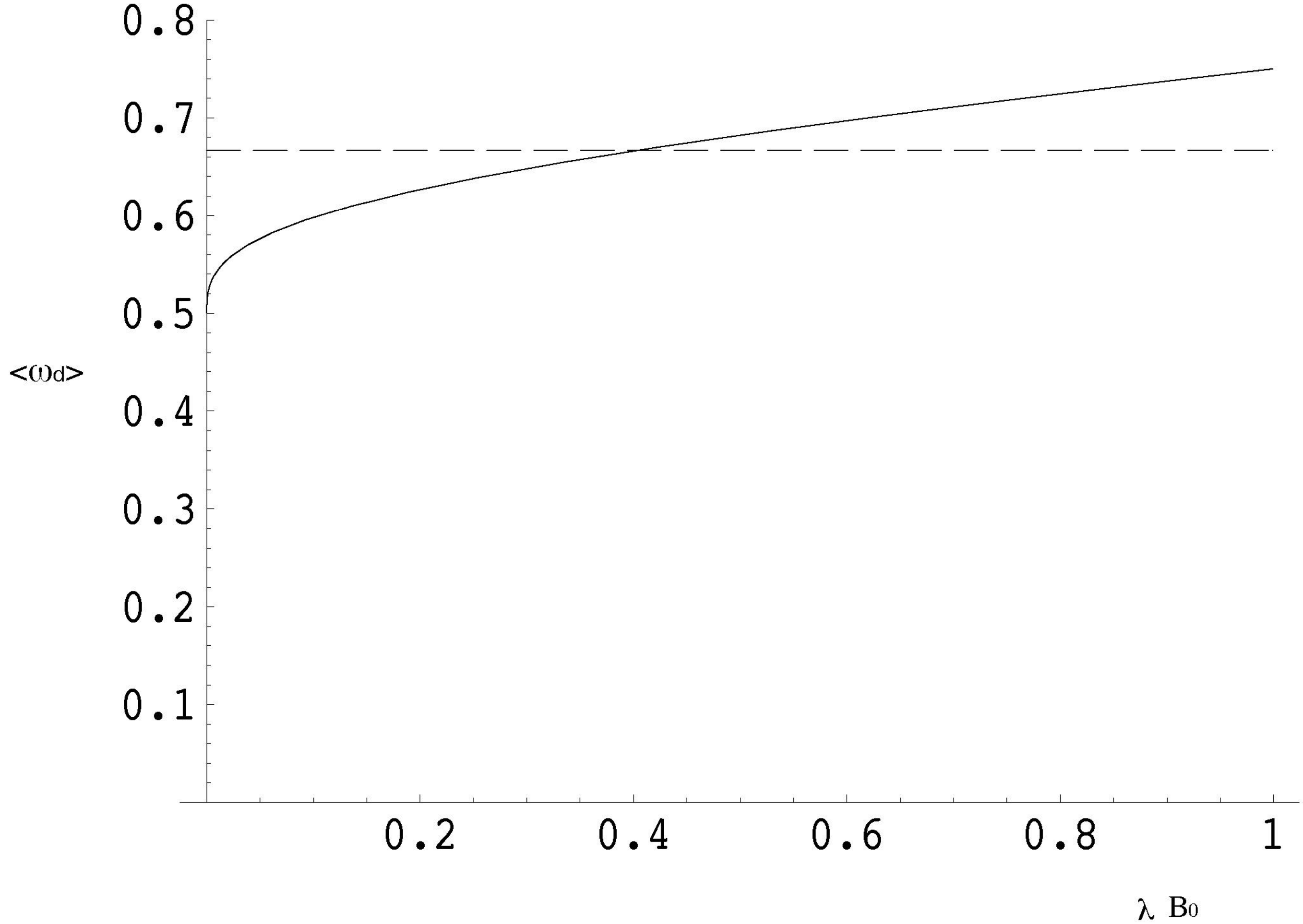
Collisionless ion response: From Eq. 1

$$\frac{\delta n_i}{n_i} = -\frac{q_i \phi}{T_i} + \frac{q_i}{T_i} \int d^3v \frac{\omega - \hat{\omega}_{*i}(1+\eta_i(\epsilon/T_i - 3/2))}{\omega - \overline{\omega}_{di}(\epsilon,\lambda)} \overline{\phi} F_0$$

$$\equiv \frac{q_i}{T_i}(-\phi + \Lambda_i(\omega, \hat{\omega}_{*i}, \hat{\omega}_{di}))$$

- Consider particle motion in a point dipole field.
  - To obtain correct MHD response approximate

$$\overline{\omega}_{di}(\epsilon,\lambda) \approx \frac{2}{3}\frac{\epsilon}{T_i}\hat{\omega}_{di} \qquad (3).$$


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
<ωd>
0.2
0.4
0.6
0.8
1
λ B0

$\overline{\omega}_{di}(\epsilon, \lambda)$ **Approximation**

- We can better approximate $\overline{\omega}_{di}(\epsilon, \lambda) \approx \frac{2}{3}\frac{\epsilon}{T_i}\hat{\omega}_{di}\,(1 + \delta(\lambda B_{min} - 0.4))$ to obtain correction to $\overline{\omega}_{di}$. Find $\delta = 0.12$.

  For $\delta \ll 1$ obtain

  $\frac{\delta n_i}{n_i} \approx -\frac{q_i \phi}{T_i} + \frac{3\tau d}{2}\frac{1-\eta}{1+\eta}\int \frac{B\overline{q_i\phi/T_i}\, d\lambda}{\sqrt{1-\lambda B}}(1-\delta(\lambda B_{min}-0.4))$

  & similar for electrons. Consider

- Consider quasi-neutrality with $\phi = \phi_0 + \delta\ \phi_1(\ell) + \cdots$ and $d = d_0 + \delta\ d_1 + \cdots$

  - Lowest order → flute solution.

  - Flux tube average of 2nd order →

$$\frac{d_1}{d_0} = \left(\frac{2}{3}\frac{B_{min}\oint d\ell/B^2}{\oint d\ell/B} - 0.4\right) \approx 0.06$$

- This yields 1 % correction in Eq. 3.

Dispersion Relation - Semi-Collisional Regime

- Include collisional electron response and apply quasi neutrality:

  $$2\phi = \Lambda_i(\Omega, d, \eta) + \langle\phi\rangle\Lambda_e^c(\Omega, d, \eta) \qquad (4)$$

  with $\Omega = \omega/\hat{\omega}_{de}$, $d = \hat{\omega}_{*e}(1+\eta)/\hat{\omega}_{de}$.

  Taking flux tube average yields: $2 = F_i + \Lambda_e^c$

  $F_i(\omega) = \int d^3v\ F_0\ \frac{\omega - \hat{\omega}_{*i}(1+\eta_i(\epsilon/T_i - 3/2))}{\omega - \frac{2}{3}\frac{\epsilon}{T}\hat{\omega}_{di}}$

- There is a flute eigenmode solution to Eq. 4.

- Dispersion relation can be written in form:

  $D(\omega) = \frac{d}{1+\eta}\left[F_1(\omega) + \eta F_2(\omega)\right] - F_3(\omega) = 0.$

- There is no marginal stability for $\omega/\hat{\omega}_{di} > 0$ and therefore no ion drift resonances. Thus have coincident real roots and $\partial D/\partial\omega = 0$.

  One can show $(F_2'F_3 - F_3'F_2) = -(3/2)(F_1'F_3 - F_3'F_1)$.

  Thus obtain

  $$(1 - \frac{3}{2}\eta)(F_1'F_3 - F_3'F_1) = 0$$

- Therefore $\omega_{crit} = 0.32\ \hat{\omega}_{de}$ and

$$d = 0.66\ \frac{1+\eta}{1-0.51\ \eta}. \qquad (5)$$

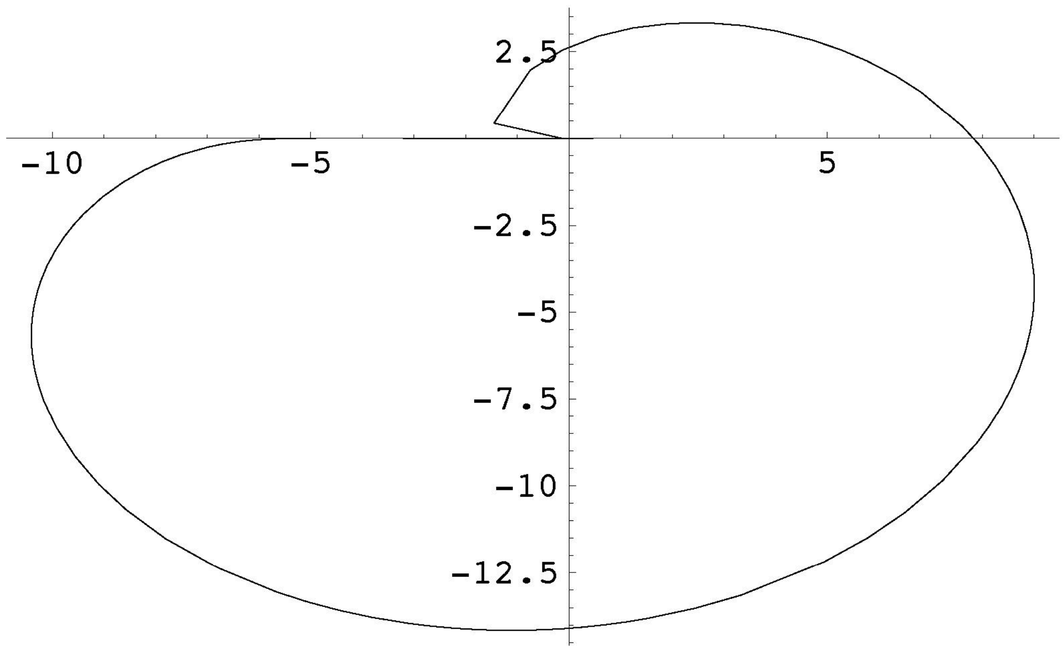
- Can evaluate stability numerically. Mathematica will evaluate error functions.
  - Nyquist plot indicates # of roots and stability.
  - Zero finder evaluates root.
  - For kinetic integrals:

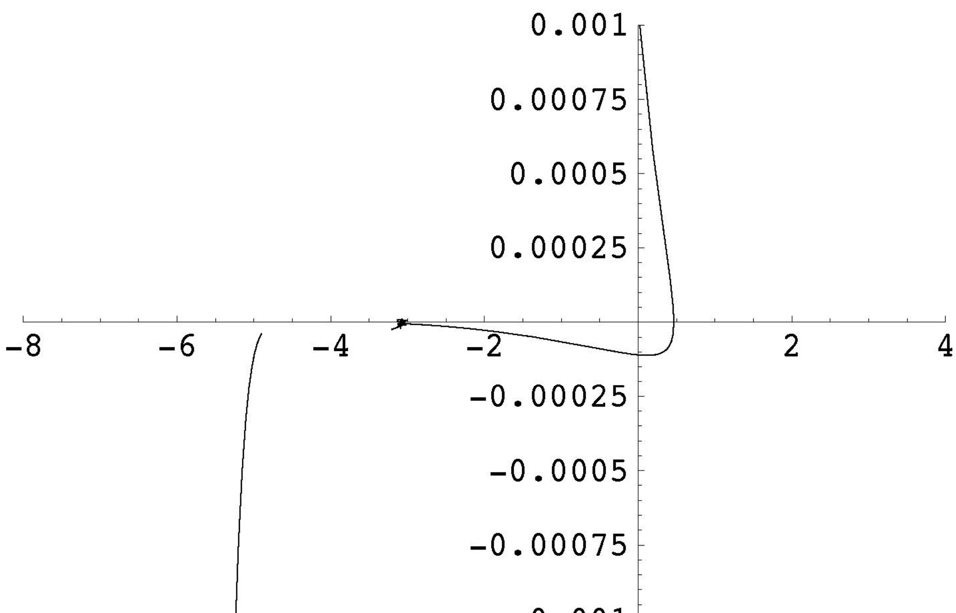
$$\frac{4}{\sqrt{\pi}} \int_0^\infty \left(\frac{x^2 e^{-x^2} dx}{y + \frac{2}{3} x^2} \right) =$$

$$T \sqrt{\frac{3\pi y}{2}} \left(-3\, e^{\frac{3\,y}{2}} + \sqrt{\frac{6}{\pi y}} + 3\, e^{\frac{3\,y}{2}}\, Erf\left(\sqrt{\frac{3}{2}}\, \sqrt{y}\right) \right)$$

NYQUIST PLOT

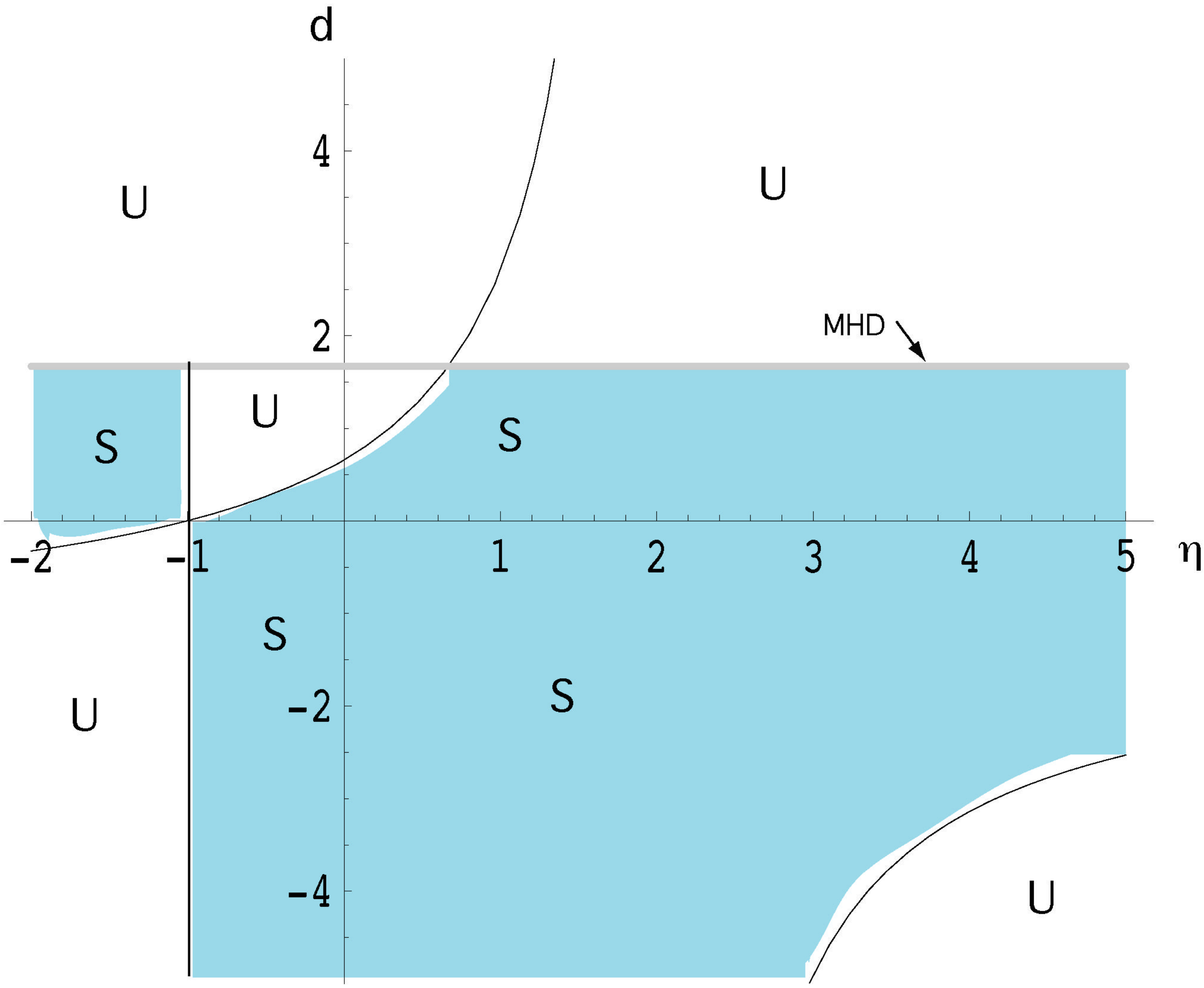
Nyquist plot for stable point; d=-2, $\eta=-0.2$





Vicinity of axis

Semi-collisional Mode



Collisionless Ions and Electrons

Ref. M.N. Rosenbluth, Phys. Fluids **11**, 869 (1968).

J. Kesner, Phys Plasmas **5**, 3675 (1998).

- Rosenbluth considered collisionless isothermal plasma ($\eta = 0$) in closed field line system. No FLR→ No MHD mode.
  - If any particles bounce in bad curvature always find an instability for $d > d_{crit}$.

    Note - In dipole all bounce in bad curvature.
- We consider arbitrary η and both good ($d < 0$) and bad ($d > 0$) curvature.
- Collisionless dispersion relation

$$2\phi = \int d^3v \frac{\omega - \omega_{*e}(1+\eta_e(\frac{\epsilon}{T_e} - \frac{3}{2}))}{\omega - \frac{2}{3}\frac{\epsilon}{T}\hat{\omega}_{de}} \overline{\phi} F_{0e}$$

$$+ \int d^3v \frac{\omega + \omega_{*e}(1+\eta_i(\frac{\epsilon}{T_i} - \frac{3}{2}))}{\omega + \frac{2}{3}\frac{\epsilon}{T}\hat{\omega}_{de}} \overline{\phi} F_{0i}$$

$$= \frac{1}{2}(\Lambda_e + \Lambda_{i0}) \int \frac{B d\lambda}{\sqrt{1-\lambda B}} \overline{\phi} \qquad (6)$$

Taking the flux tube average can obtain $2 = \Lambda_e + \Lambda_{i0}$

Substitute into (6), take flux tube avg to obtain:

$$\oint \frac{d\ell}{B} \int \frac{B d\lambda}{\sqrt{1-\lambda B}} (\phi^2 - \overline{\phi}^2) = \int \tau_b \, d\lambda (\overline{\phi^2} - (\overline{\phi})^2) = 0 \ .$$

Since $\overline{\phi^2} - \overline{\phi}^2 \geq 0$ obtain flute like, i.e. $\phi = \phi_0$ to order $k_\perp^2 \rho_i^2$.

- Consider finite $k_\perp^2 \rho_i^2 \ll 1$. Obtain:

$$-2\phi + \Lambda_i + \Lambda_e \langle \phi \rangle + k_\perp^2 \rho_i^2 \ \Lambda_{1i}(\ell) = 0$$

We can expand $\phi = \phi_0 + t\phi_1 + \ldots$ and $\omega = \omega_0 + t \ \omega_1 + \ldots$. Then:

$$-2\phi_0 + \Lambda_i(\phi_0) + \Lambda_e \langle \phi_0 \rangle = 0$$

and to next order:

$$-2\phi_1 + \Lambda_i(\phi_1) + \Lambda_e \langle \phi_1 \rangle + \Lambda_{1i}(\ell, \langle \phi_0 \rangle) + \frac{\partial \Lambda_i(\phi_0)}{\partial \omega} \omega_1 + \frac{\partial \Lambda_e}{\partial \omega} \omega_1 \langle \phi_0 \rangle = 0.$$

Integrating $\oint d\ell / B$

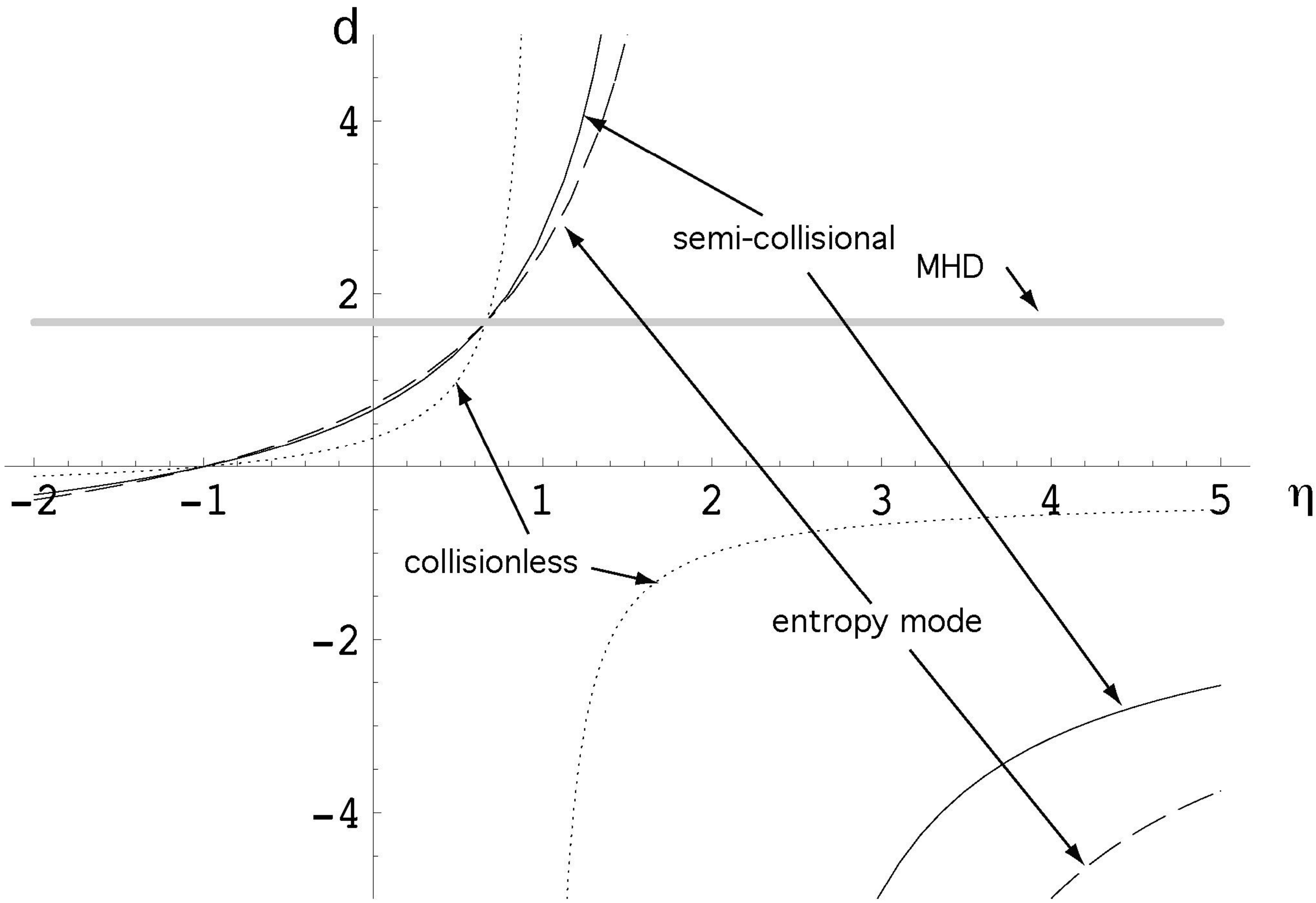
$$\omega_1 \frac{\partial}{\partial \omega} [\Lambda_i(\omega_0) + \Lambda_e(\omega_0)] + \oint \frac{d\ell}{B} \Lambda_{1i}(\ell, \omega \equiv \omega_0) = 0.$$

This gives the shift in ω away from ω_0, caused by $k_\perp^2 \rho_i^2 \neq 0$.

- Following Rosenbluth look for marginality condition with Re[ω]= Im[ω]=0.

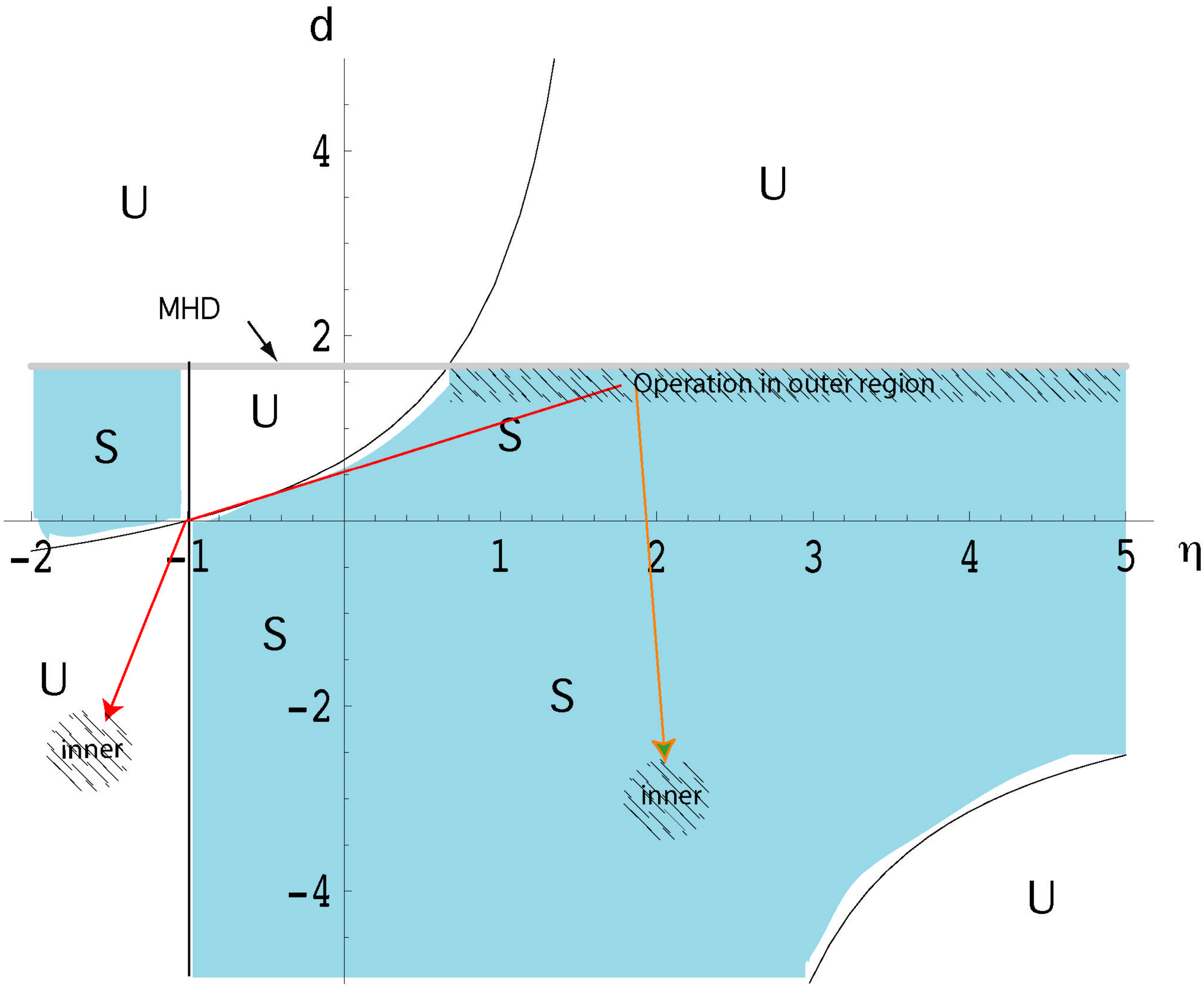
$$d = \frac{1}{3}\left[\frac{1+\eta}{1-\eta}\right]. \tag{6}$$

- Stability boundary is *similar* but *more restrictive* than collisional case.



d
4
2
-2
-4
-2
-1
1
2
3
4
5
η
semi-collisional
MHD
collisionless
entropy mode

The pressure profile maps out a trajectory in (d,η) space.



Semi-collisional Mode

Conclusions

- 2 modes are present; MHD-like and drift mode
  - MHD mode stable when $d < 5/3$.
  - Drift mode driven by bad curvature ($d > 0$) and profile, i.e. η, effects.
- Collisionality is stabilizing; collisionless modes show larger area of instability.
- Levitated dipole
  - In region between the pressure peak and the wall $\nabla T < 0$, $\nabla n_e < 0$ and therefore $\eta > 0$.
  - At the pressure peak $d = 0$ and $\eta = -1$.
  - Between the pressure peak and the internal coil

    LDX: $\nabla T > 0$, $\nabla n_e > 0$ and $d < 0$, $\eta > 0$.

    Reactor: $\nabla T > 0$, $\nabla n_e < 0$ and $d < 0$, $\eta < -1$.

Sign-up sheet.

Poster and paper will be available on the www at www.psfc.mit.edu/ldx/