ABSTRACT

Magnetic Diagnostics in LDX

The Levitated Dipole Experiment (LDX) will investigate the equilibrium and stability of a high-beta plasma confined in a dipolar magnetic field created by a levitated superconducting current ring. One of the principal objectives of the experiment is to understand the relationship between plasma equilibrium and stability in a dipole geometry. The equilibrium pressure profile will be found by iteratively fitting the Grad-Shafranov equation to the magnetic measurements obtained from multifarious magnetic sensors. Equilibrium fields will be measured with pick-up coils and flux loops placed outside the vacuum vessel. Hall probes have been attached to each of the coils to supplement the coil measurements. A toroidal array of poloidally oriented Mirnov coils will be placed inside the vessel to detect and characterize fast (MHz) plasma fluctuations caused by instabilities. The current progress on the diagnostics will be presented along with their test results. Model equilibrium and simulated reconstructions will also be shown.

WHAT MAGNETIC DIAGNOSTICS WILL ACCOMPLISH

- Deduce the current and pressure profiles via the reconstruction scheme
- Obtain information regarding plasma shape and position
- Extract plasma beta profile
- In conjunction with density diagnostics, construct the temperature profile to cross-check with that obtained from x-ray diagnostics
- Get the total stored (kinetic + magnetic) energy density

MAGNETIC DIAGNOSTICS OVERVIEW FOR THE LDX

- 18 poloidal B-coils to measure orthogonal fields at 9 different locations
- 9 poloidal flux-loops
- A set of Hall probes to supplement the coil measurements and to use in cross-checking purposes
- A set of Mirnov coils to measure fast plasma fluctuations

EQUILIBRIUM RECONSTRUCTION ITERATION MECHANISM

Iterative Grad-Shafranov Equation is used along with the pressure model:

$$\square^{\square}\square^{k+1} = -\square_0 R J_{\square}^{k}(R, \square^{k}, \square_n^{k})$$

$$J_{\square}^{k} = RP'(\square^{k}, \square_n^{k})$$

$$P'(\square^{k}, \square_n^{k}) = \square_{n=1}^{n_p} \square_n^{k}\, \square_n(\square^{k})$$

In addition, the $\square_n^k$'s are found in such way as to minimize the merit function $\square$:

$$\square_n^{k+1} = [L^{\dagger}]_{nj}^{k}(M_j - C_j^{k}) + \square_n^{k}$$

- L.L. Lao, et al., Nucl. Fusion **30**, 1035 (1990).

THE MERIT FUNCTION AND ITS ROLE IN THE ITERATION SCHEME

The merit function $\Box^2$ is defined as following:

$$\Box^2 = \Box_{j=1}^{n_m} \frac{(M_j - C_j^k)^2}{\Box_j^2} \quad \text{where}$$

M_j	=	measurement value at the j^{th} detector
C_j^k	=	calculated value from $\Box^k$ or J^k at the j^{th} detector position
$\Box_j$	=	measurement error

Since $C_j^k = C_j^k(\Box^k, \Box_n^k)$ we can choose $\Box_n^k$ in each iteration so as to minimize $\Box^2$

PROSPECTIVE PRESSURE MODELS

One of the simplest and most commonly used pressure models is the polynomial:

$$\square_n(\square) = \square^n \qquad \text{where } \square \equiv \frac{\square - \square_{FCFS}}{\square_{LCFS} - \square_{FCFS}} \quad \text{(Normalized Flux)}$$

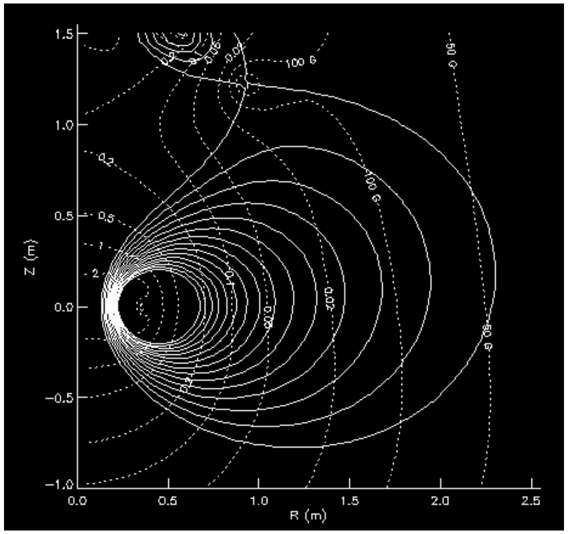
$$\boxed{P'(\square,\square_n) = \mathop{\square}_{n=1}^{n_p} \square_n \square^n}$$

In LDX, the adiabatic compressibility condition allows for a potentially better model of the following form:

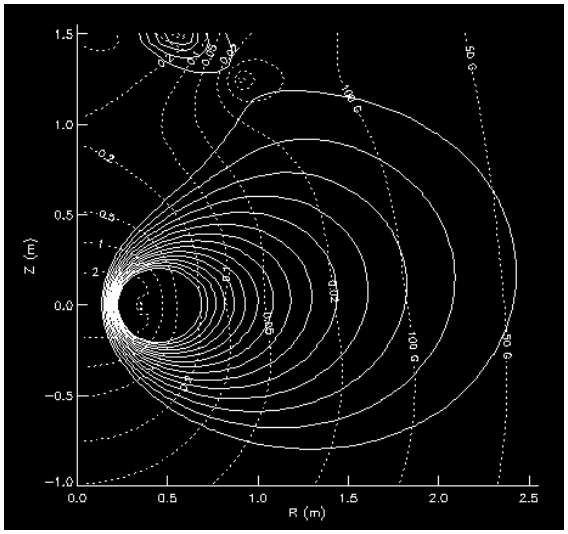
$$P(\square,\square_p,P_{edge},g) = \begin{cases} P_{edge}[V(\square)/V_{edge}]^{-g} & \text{for } \square > \square_p \\ P_{edge}[V(\square)/V_{edge}]^{-g}\sin^2[(\square/2)(\square/\square_p)^2] & \text{for } \square < \square_p \end{cases}$$

- D.T. Garnier, et al., Phys. of Plasmas 6, 3431 (1999).

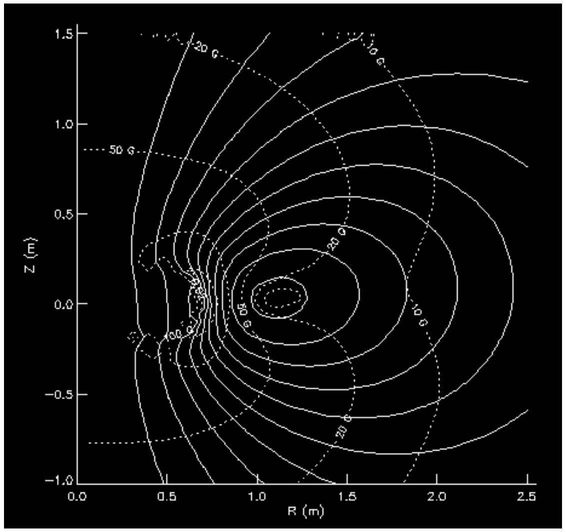
TYPICAL FIELD CONFIGURATIONS



Vacuum Field



Low Beta Plasma Field



Difference Field

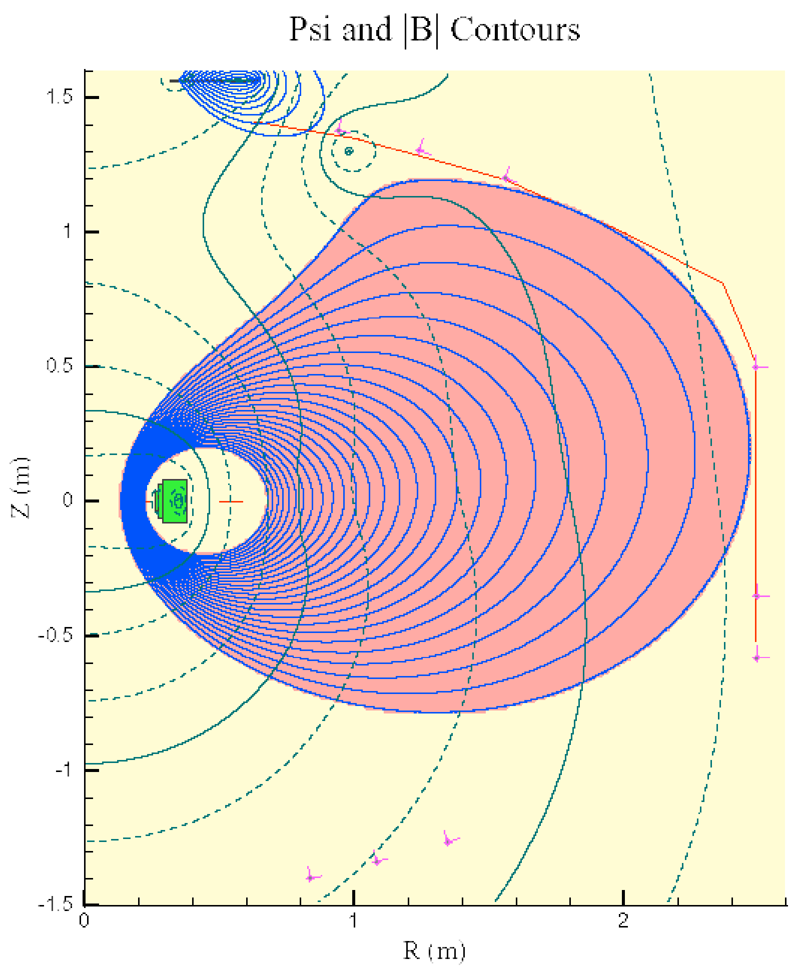
Assumed Conditions:

$P_{edge} = 1Pa$

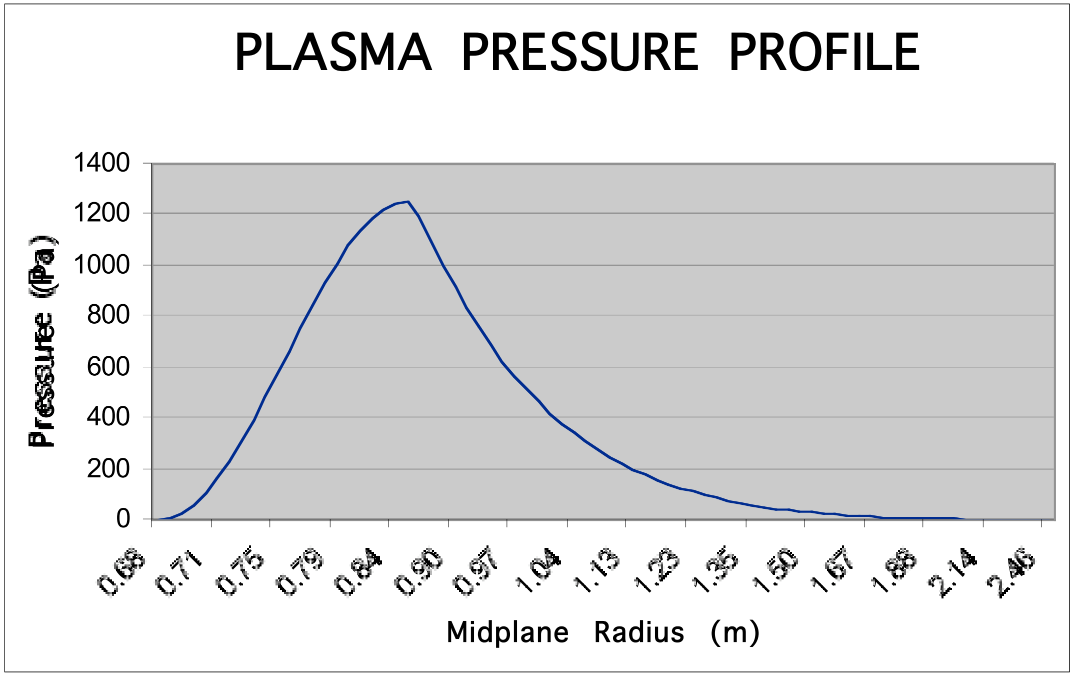
$R_{peak} = 0.71m$

$g = 5/3$

SAMPLE PLASMA SHAPE ALONG WITH ITS PRESSURE PROFILE



$P_{edge} = 1Pa$
$R_{peak} = 0.85m$
$g = 5/3$



SENSOR LOCATION OPTIMIZATION

<u>Fundamental Concept</u>: There is a functional relationship between plasma parameters and external magnetic measurements.

→ $\mathbf{m} = \mathbf{F}(\mathbf{p})$, where

m is an m-dimensional vector of different types of measurements at different positions

p is an n-dimensional vector of plasma parameters

F: $\mathbf{R}^n \rightarrow \mathbf{R}^m$ is the response function

MEASUREMENT AND PARAMETER VECTORS

Examples of components of **m** and **p**:

$m_1 \ldots m_i$ = B_p-coil measurements at i different positions
$m_{i+1} \ldots m_{2i}$ = flux loop measurements at i different positions
and so on…

p_1 = edge pressure at midplane
p_2 = peak pressure
p_3 = current density at the peak pressure position
p_4 = average beta
and so on…

OPTIMIZATION STRATEGY

Goal: Find the function **F** that relates plasma parameters to the measurements. Then the components of $\square_{\mathbf{m}}\mathbf{F}^{-1}$ will give the sensitivity of the different types of measurements at different positions to changes in the plasma parameters.

Method: Execute multiple runs of the equilibrium code to get many sets of **m**'s and **p**'s. Linearize **F** about multiple points in the parameter space in order to obtain a set of linearized **F**'s that are valid in a small region in the parameter space. Solve for each of the linearized **F**'s using the **m**'s and **p**'s corresponding to the region where the linearized **F**'s are valid.

EXPANSION OF THE FUNCTION F

Linearization of **F**:

$$\mathbf{m} = \mathbf{F}(\mathbf{p}) \approx \mathbf{F}(\mathbf{p}_0) + (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet (\mathbf{p} - \mathbf{p}_0)$$

$$= \mathbf{F}(\mathbf{p}_0) - (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet \mathbf{p}_0 + (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet \mathbf{p}$$

$$\equiv \mathbf{k} + \mathbf{R} \bullet \mathbf{p}, \text{ where} \qquad \mathbf{k} \equiv \mathbf{F}(\mathbf{p}_0) - (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet \mathbf{p}_0$$

$$\mathbf{R} \equiv (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0}$$

The problem of finding the function **F** is now reduced to finding the elements of **k** and **R** for each small subdomain of the parameter space.

COUNTING THE UNKNOWNS

- Notice that **k** is an m-element vector and **R** is an $m \times n$ matrix.

- We need $m + m \times n$ independent equations in order to solve for all the elements of **k** and **R**.

- A single set of **m** and **p** gives m independent equations.

We require n+1 sets of **m** and **p**, i.e. $(\mathbf{m}^1, \ldots, \mathbf{m}^{n+1})$, $(\mathbf{p}^1, \ldots, \mathbf{p}^{n+1})$, to obtain the elements of **k** and **R**.

APPLICATION TO LDX

Specific Example (LDX plasma):

The plasma is characterized by the following three parameters:

$$\mathbf{p} = [p_1, p_2, p_3] = [p_{edge}, r_{peak}, f_{crit}]$$

We are looking at 3 different types of measurements at 43 possible locations on the vacuum vessel:

$$\mathbf{m} = [m_{BT1},\ldots, m_{BT43}, m_{BN1},\ldots, m_{BN43}, m_{F1},\ldots, m_{F43}]$$

where m_{BTi} = B_p-coil measurement in the tangential direction
m_{BNi} = B_p-coil measurement in the normal direction
m_{Fi} = Flux-loop measurement

NUMBER OF NECESSARY EQUATIONS

Specific Example (cont'd):

- Notice that m = 129 and n = 3 → 516 unknowns!
- We need 3 + 1 = 4 sets of **m**'s and **p**'s to determine the unknowns
- One of the sets can be used to eliminate **k** and the remaining 3 can be used to solve for **R**

We have:

$$\mathbf{m}^1 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^1$$
$$\mathbf{m}^2 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^2$$
$$\mathbf{m}^3 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^3$$
$$\mathbf{m}^4 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^4$$

→

$$(\mathbf{m}^1 - \mathbf{m}^4) = \mathbf{R} \bullet (\mathbf{p}^1 - \mathbf{p}^4)$$
$$(\mathbf{m}^2 - \mathbf{m}^4) = \mathbf{R} \bullet (\mathbf{p}^2 - \mathbf{p}^4)$$
$$(\mathbf{m}^3 - \mathbf{m}^4) = \mathbf{R} \bullet (\mathbf{p}^3 - \mathbf{p}^4)$$
$$\mathbf{k} = \mathbf{m}^4 - \mathbf{R} \bullet \mathbf{p}^4$$

THE INVERSE RESPONSE MATRIX

Now, concatenate these equations to form a larger system:

$$\begin{bmatrix} \mathbf{m}^1 - \mathbf{m}^4 \\ \mathbf{m}^2 - \mathbf{m}^4 \\ \mathbf{m}^3 - \mathbf{m}^4 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & & \\ & \mathbf{R} & \\ & & \mathbf{R} \end{bmatrix} \bullet \begin{bmatrix} \mathbf{p}^1 - \mathbf{p}^4 \\ \mathbf{p}^2 - \mathbf{p}^4 \\ \mathbf{p}^3 - \mathbf{p}^4 \end{bmatrix}$$

We can solve this system for **R** then use SVD to obtain $\mathbf{R}^\dagger$.

Notice:

- $\mathbf{R}^\dagger \approx (\partial_{\mathbf{m}}\mathbf{F}^{-1})^T$ gives a measure of sensitivity.
- $\mathbf{R}^\dagger$ is valid only for ranges in the vicinity of $\mathbf{p}^1$, $\mathbf{p}^2$, $\mathbf{p}^3$, and $\mathbf{p}^4$.

THE MERIT PARAMETER

$$\mathbf{R}^{\dagger} = \begin{bmatrix} \partial p_1/\partial m_1 & \ldots & \partial p_1/\partial m_{129} \\ \partial p_2/\partial m_1 & \ldots & \partial p_2/\partial m_{129} \\ \partial p_3/\partial m_1 & \ldots & \partial p_3/\partial m_{129} \end{bmatrix}$$

Measurement k would be sensitive to parameter j if $\partial p_j/\partial m_k$ is small. Hence, define the measure as follows:

$$M_k \equiv \sqrt{(\bar{m}_k/\bar{p}_1)^2(\partial p_1/\partial m_k)^2 + (\bar{m}_k/\bar{p}_2)^2(\partial p_2/\partial m_k)^2 + (\bar{m}_k/\bar{p}_3)^2(\partial p_3/\partial m_k)^2}$$

where $\bar{m}_k \equiv \sum (m_k^i/4)$

$\bar{p}_j \equiv \sum (p_j^i/4)$ $\quad 1 \le i \le 4$

EXAMPLE CALCULATION

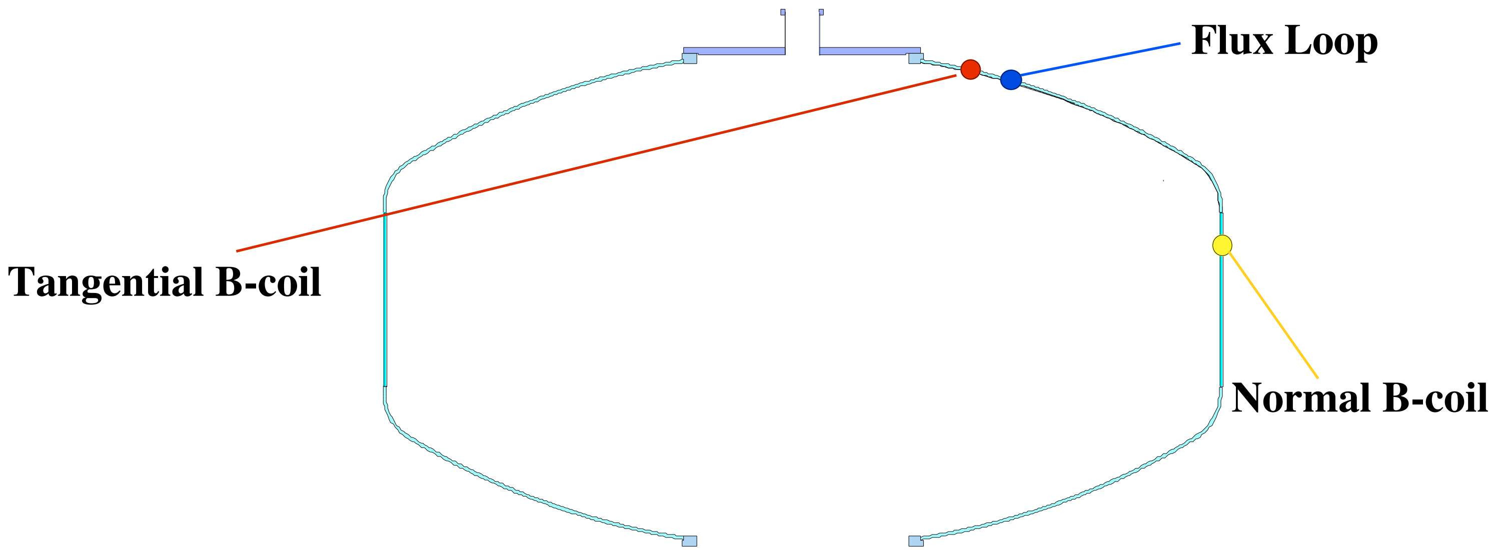
Let $\mathbf{p}^1 = [1.0 \text{ Pa}, 0.65 \text{ m}, 1.0]$

$\mathbf{p}^2 = [1.2 \text{ Pa}, 0.65 \text{ m}, 1.0]$

$\mathbf{p}^3 = [1.0 \text{ Pa}, 0.67 \text{ m}, 1.0]$

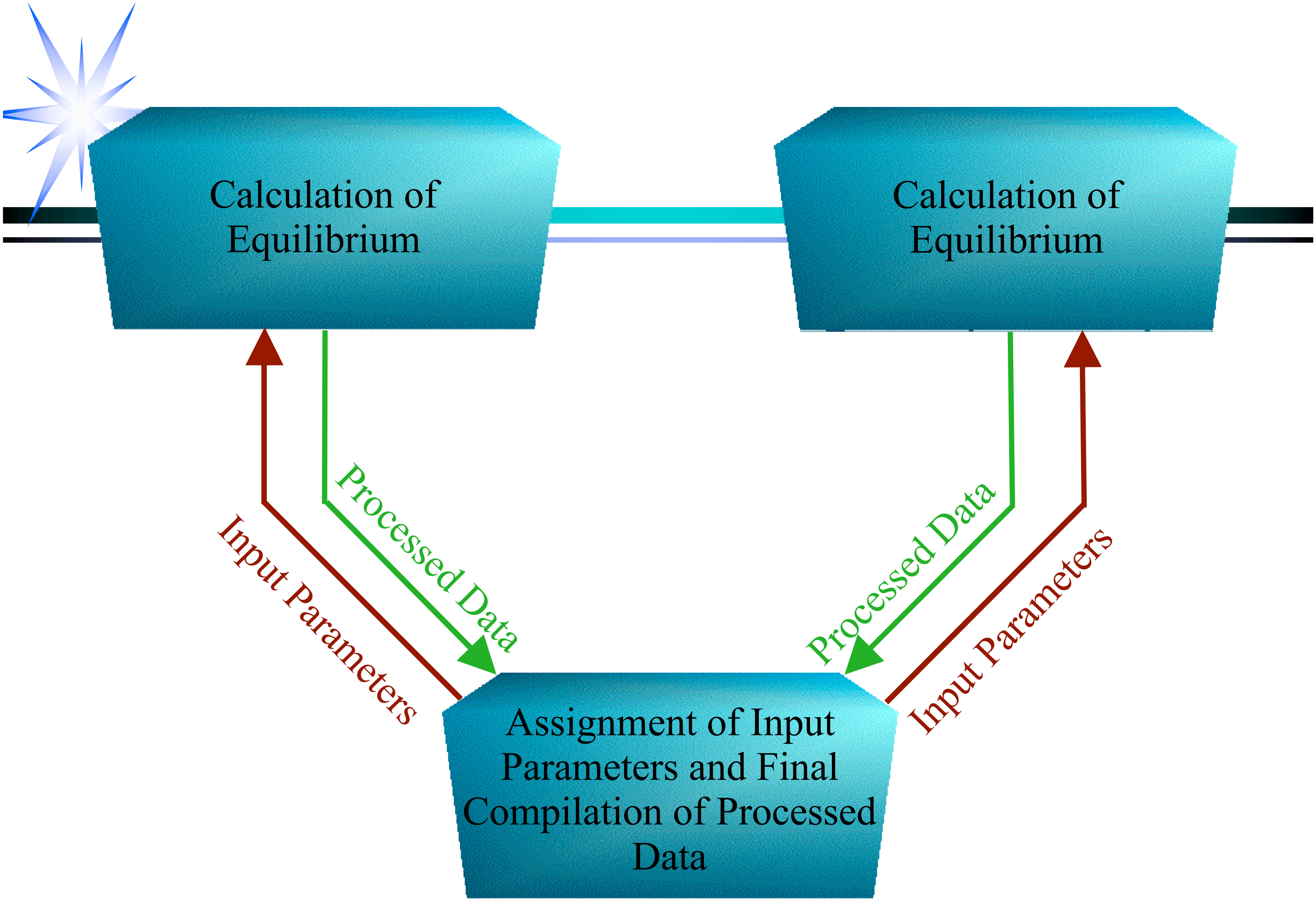
$\mathbf{p}^4 = [1.0 \text{ Pa}, 0.65 \text{ m}, 1.2]$

Then the measure is minimized for the normal B-coil, tangential B-coil, and flux loop at the following positions:

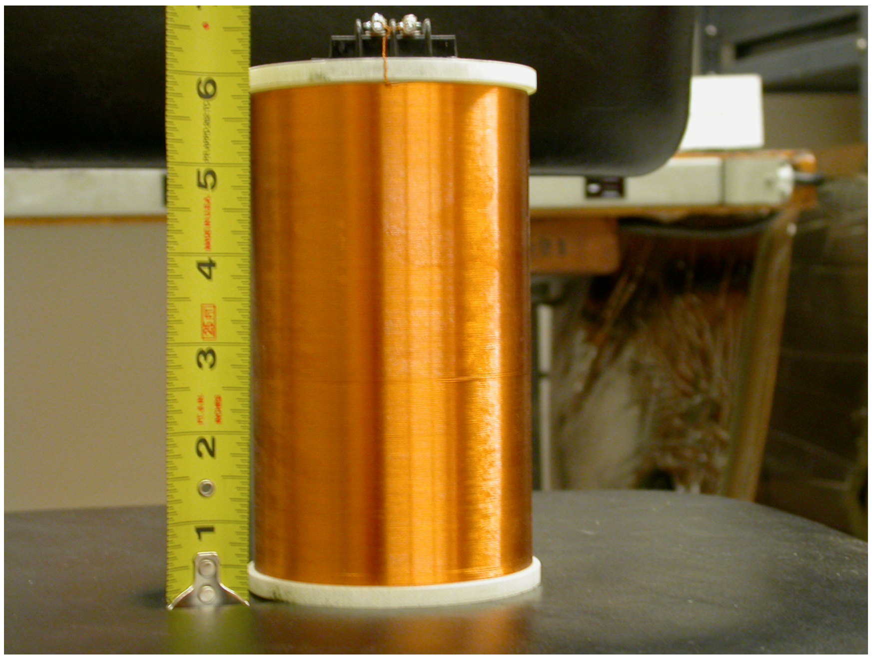


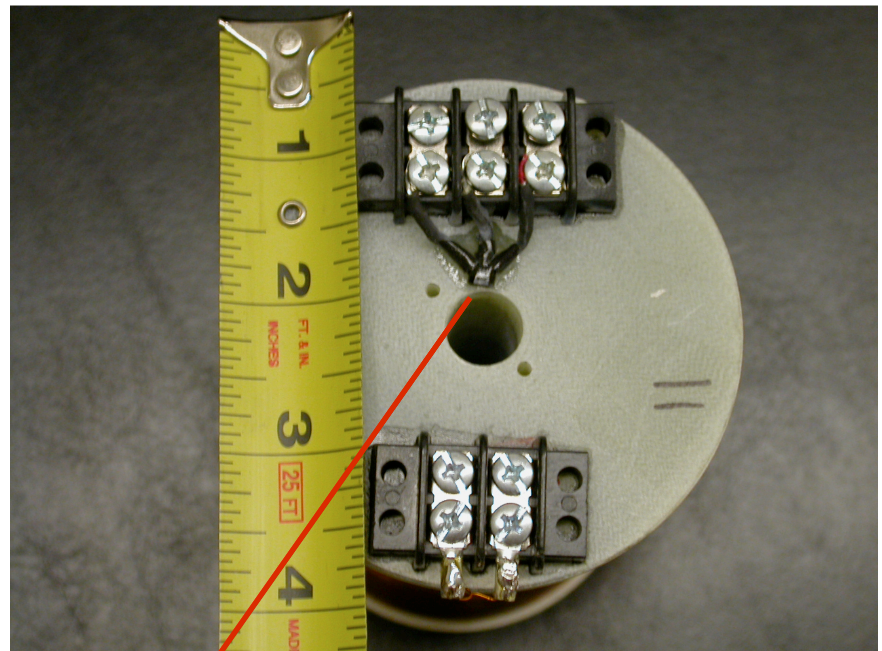
AUTOMATED GENERATION OF INPUT DATA

- Need to execute ~ 2000 plasma equilibrium runs
- Software has been written to automate the process
  - Generates sets of all possible permutations of chosen input parameters
  - Sends sets of input parameters to remote computers which compute the plasma equilibria
  - Extracts necessary data from the outputs of the plasma equilibrium code and consolidates the data into a form that can be used as inputs to the analysis code


Calculation of Equilibrium
Calculation of Equilibrium
Processed Data
Input Parameters
Processed Data
Input Parameters
Assignment of Input Parameters and Final Compilation of Processed Data

LDX B_P-COIL AND HALL SENSOR





Hall Probe

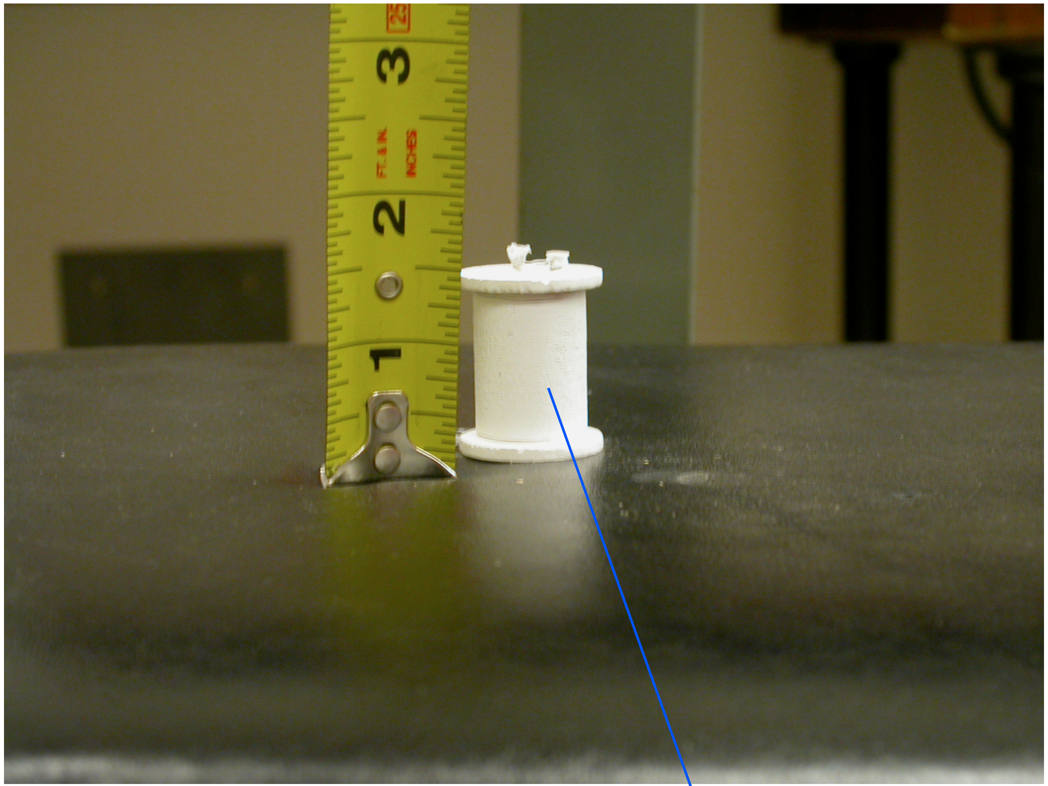
B_p-Coil Specs:

- NA ~ 5 m^2
- Sensitivity: 500 mV/G
  (connected to a 1 ms RC integrator)

Hall Sensor Specs:

- Field Range: +/- 500 G
- Sensitivity: 5 mV/G

LDX MIRNOV COIL



Boron Nitride Coated

NA: $\sim 0.06\ m^2$

L/R_0: $\sim$ 50 ps

f_0: ~20 GHz

- Directly measures dB/dt
- Placed inside the vessel
- Measures fluctuations in the microsecond range