Theory of Plasma Confinement in a Levitated Dipole

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Requirements for "ideal" fusion confinement device.

- MHD instability does not destroy plasma, i.e. no disruptions
- Steady state operation
- High β for economic utilization of field
- High τ_E (before ignition)

  Ignition in small device

  Advanced fuel (DD, D-He) possibility
- Low τ_p for ash removal
- Low divertor heat load:

  Plasma outside of TF coils $\rightarrow$ large flux expansion.
- Circular, non interlocking coils.

Levitated dipole may fulfill these requirements if physics "works" and technology does not introduce new show-stoppers.

Is B_T necessary for toroidal confinement?

- B_p only: equilibrium but MHD unstable (i.e. FRC)

Two solutions:

(tok, stell, RFP etc)	Levitated dipole
Add $B_T \rightarrow$ MHD stable from well and shear.	Levitated ring $\rightarrow$ MHD stable from compressibility
$\beta << 1 \quad (\beta_p \sim 1)$	$\beta > 1$ when $p' < p'_{crit}$
Drifts off flux surfaces $\rightarrow$ neoclassical effects	No drift off flux surfaces
particles trapped in bad curvature $\rightarrow$tpm's	No tpm's but drift modes possible near ring.

Important Differences

Magnetic shear $\rightarrow$ No convective cells	Can have convective cells, but without energy transport.

Critical Issues

Divertor Steady state Disruptions	Internal ring neutron heat Low power density

Interchange Stability; Rosenbluth-Longmire†

- Closed field line configuration can have "absolute" well when exchange of flux tubes causes internal plasma energy (work+compressibility) to increase.

  Assume equation of state: $p/\rho^\gamma = f(\psi)$.

  $\Delta E_p = \delta p \delta_V + \gamma p \frac{\delta V^2}{V} = \delta V \delta S / V^\gamma$.

  $V = d(Vol)/d\psi = \oint dl/B$, S=entropy=p$V^\gamma$

- For $\delta S > 0$ any exchange of flux tubes will increase plasma energy and damp perturbation.
  - When $\nabla p/p < \gamma \nabla V/V$, MHD perturbation will damp and vica versa.
  - For $S = const$, $p_{core}/p_{edge} = (V_{edge}/V_{core})^\gamma$.
  - For Dipole $p_{crit} \propto V^{-\gamma} \rightarrow p_{crit} \propto r^{-20/3}$.
  - Since $B^2 \propto r^{-6}$, $\beta = 2\mu_0 p_{crit}/B^2 \propto r^{-2/3}$ β only decays slowly.

(Microscopically compressibility comes from conservation of adiabatic invariants, μ and J.)

† *Rosenbluth and Longmire, Ann Phys.* **1** *(1957) 120.*

Some Early References

A. Hasegawa, Comments on Plasma Physics and Controlled Fusion, **1**, (1987) 147.

A. Hasegawa, L. Chen and M. Mauel, Nuclear Fus. **30**, (1990) 2405.

E. Teller, A. Glass, T.K. Fowler et al., Fusion Technology **22**, (1992) 82.

Some Recent Dipole Theory References

(MIT, Columbia, IFS, UCLA, UMd, Kurchatov)

- MHD
  - 1a. Garnier, Kesner, Mauel, Phys Pl **6** (1999) 3431.
  - 2a. Krasheninnikov, Catto, Hazeltine, PRL **82** (1999) 2689, and others.
  - 3a. Simakov, Catto, Krasheninnikov, Ramos, Phys Pl **7** (2000)2526.
- Kinetic theory (Electrostatic)

1b. Kesner, Phys Plasmas **7** (2000) 3837.

2b. Simakov, Catto, Hastie Phys Plasmas **8**, 4414 (2001).

3b. Kesner, Hastie, to be published in Phys Pl (2002).

- Kinetic theory (Electromagnetic)

1c. V. Pastukhov and A. Yu. Sokolov, Nuc. Fusion 32 (1992) 1725.

2c. Wong, Horton, Van Dam, Crabtree, Phys Pl (**8** (2001). 2415.

3c. Simakov, Hastie, Catto, Phys Pl **9** (2002) 201.

- Non-linear

1d. Tonge, Huang, Leboeuf, Dawson, 2001 APS DPP (LP1059).

2d. Pastukhov and Chudin, Plasma Physics Report, **27**, (2001) 963.

3d. Rey and Hassam, Phys. Plasmas **8**, 5151 (2001).

MHD: Levitated Dipole

- Consider plasma confined in the field of "floating" ring:

  Similar to planetary magnetosphere but field lines close through hole in ring $\rightarrow$ losses across the field.

- From the point of view of MHD keep in mind:

  No rotational transform, $\vec{B} = \vec{B}_p$

  No shear

  Closed field lines (like multipoles)

- Systems with non-rational flux surfaces obtain stability from "average" well and from shear. Dipole stabilized by "compressibility"

  Early Reference:

  Bernstein, Frieman, Kruskal, Kulsrud, Proc. R. Soc. London, Ser. A, **244** *(1958) 17.*

MHD Equilibrium

- No rotational transform: $\vec{J} = J_\zeta \vec{e}_\zeta$.

  Grad-Shafranov equation becomes:

$$\triangle^* \psi = -\mu_0 R J_\zeta = -\mu_0 R^2 \frac{dp}{d\psi}$$

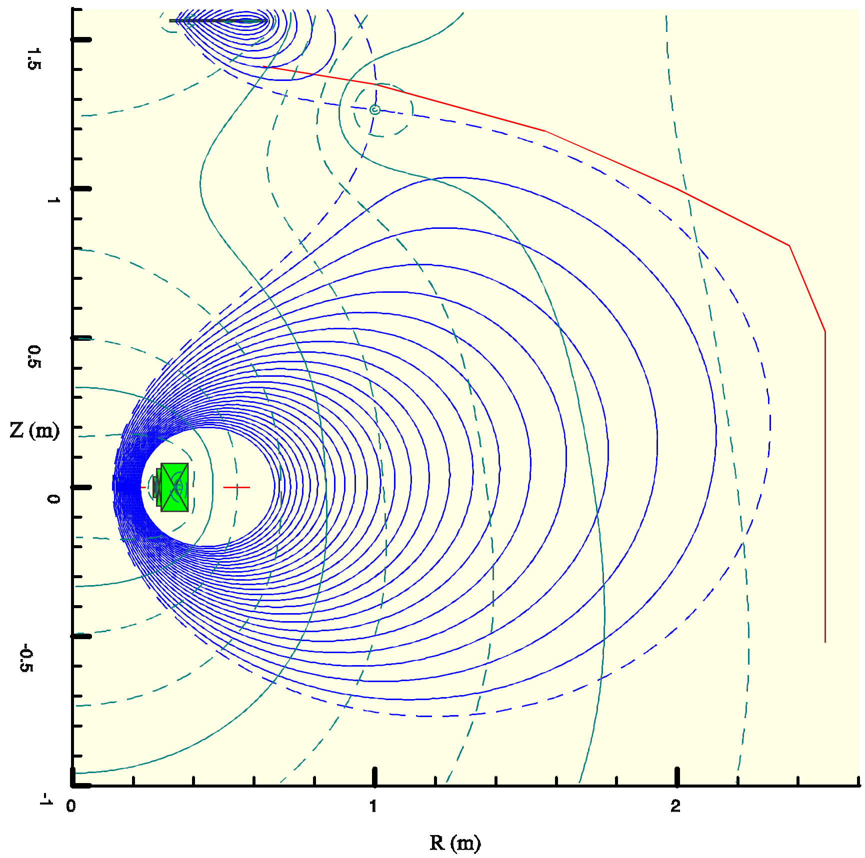
- Solved by dipole equilibrium code using multi-grid relaxation method for arbitrary beta [1].

  Use $S = const$ pressure profile ($p \propto V^{-\gamma}$)

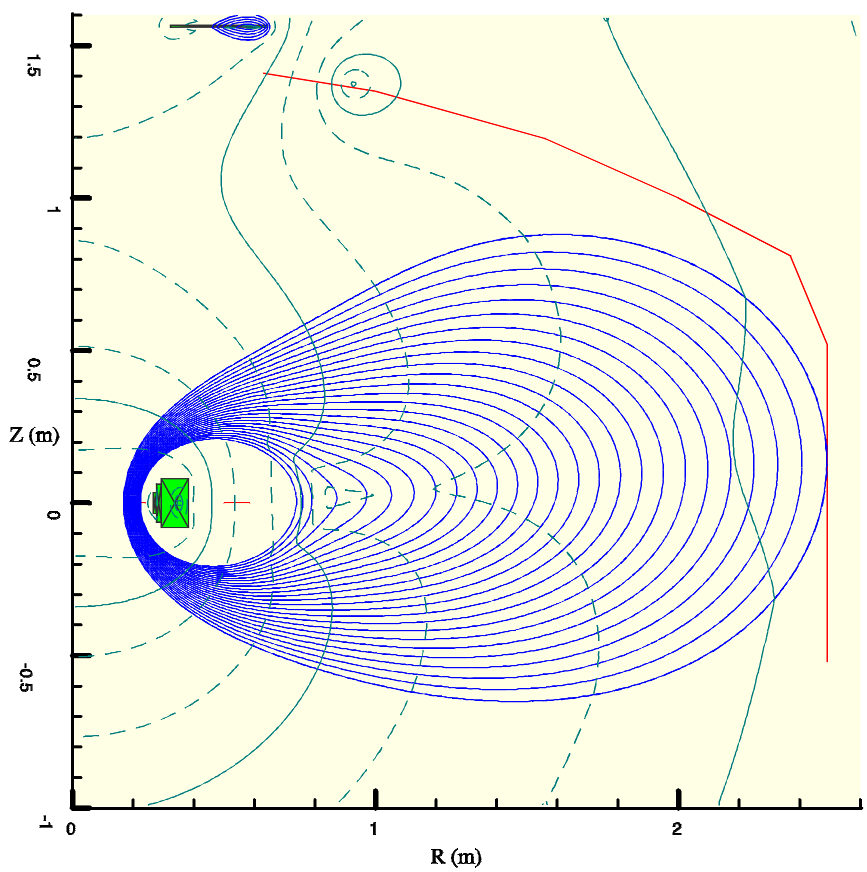
- Analytic solution also found for point dipole and sub critical pressure profile[2].

  Pressure profile chosen such that β(Z=0)=constant.

LDX Equilibria



Vacuum field



β_{max}=25

Stability of High-n Ballooning Modes

From MHD Energy Principle can show:

* Curvature drive is destabilizing between pressure peak and outer wall.
* Plasma and magnetic field compressibility and bending always stabilizing.

For interchange modes we obtain the requirement:

$$\frac{p_\psi}{p} < 2\gamma \frac{\langle \kappa_\psi \rangle}{1+\gamma\langle\beta\rangle/2}$$

- Minimize δW to obtain ODE for ballooning stability.

  (The properties of closed field line balloon eq was discussed by Bernstein et al (1958).)

- **Ballooning stability**
  - For LDX equilibrium at marginal interchange pressure ($p \propto V^{-\gamma}$) and high β ($\beta_{max} \gg 1$) have found that that the lowest order odd mode and all higher modes are stable [Garnier et al].
  - Semi-analytic point-dipole equilibrium with sub-critical pressure gradient finds same stability for $\beta \to \infty$.

- Bernstein et al showed the lowest order even mode is stable when the interchange mode is stable. (At marginal stability interchange and ballooning modes coalesce.)

Conclusion: Ideal MHD

- Equilibrium solved analytically and numerically

  Dipole exhibits equilibrium all β.

- Maximum β (for a given radial extent of plasma) obtained by choosing equilibria that are marginally interchange stable

- High β equilibria found to be stable to high-n ballooning modes.

When ∇p exceeds ∇p_{crit} convective cells form.

Some early references on convective cells:

Dawson and Okuda, PRL **27** *(1971) 491.*

Navratil et al, PF **20** *(1977) 157.*

- **With closed field lines pressure and electric potential tend to be constant on field lines. This leads to equilibrium variation in flux and toroidal angle.** Convective cells result from:

  → Non-symmetric fueling & heating

  → Instability, i.e. $p' > p'_{crit}$

- Pastukhov [2d] solved non-linear fluid equations including slow (equilibrium evolution) and fast (MHD) time scales. Results:

  - Inner plasma (coil to pressure peak) has $\nabla S > 0$ and is stable.

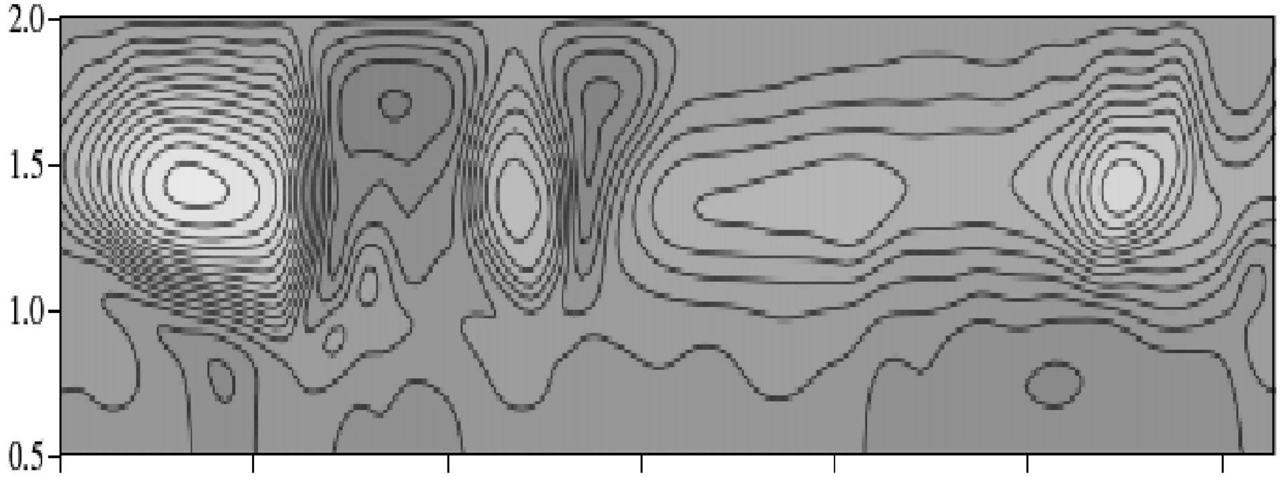
  - Outer plasma exhibits equilibrium with large convective cells → non local transport.

    $S \sim 0$ from pressure peak to close to plasma edge edge. Particle convection with small energy transport.

    Pastukov solutions are *surprisingly H-mode like.*

Pastukhov solved non-linear fluid equations for hard-core pinch
[Pastukhov, Chudin, Pl Phys Reports 27(2001) 907]

Pressure peak



Potential contours

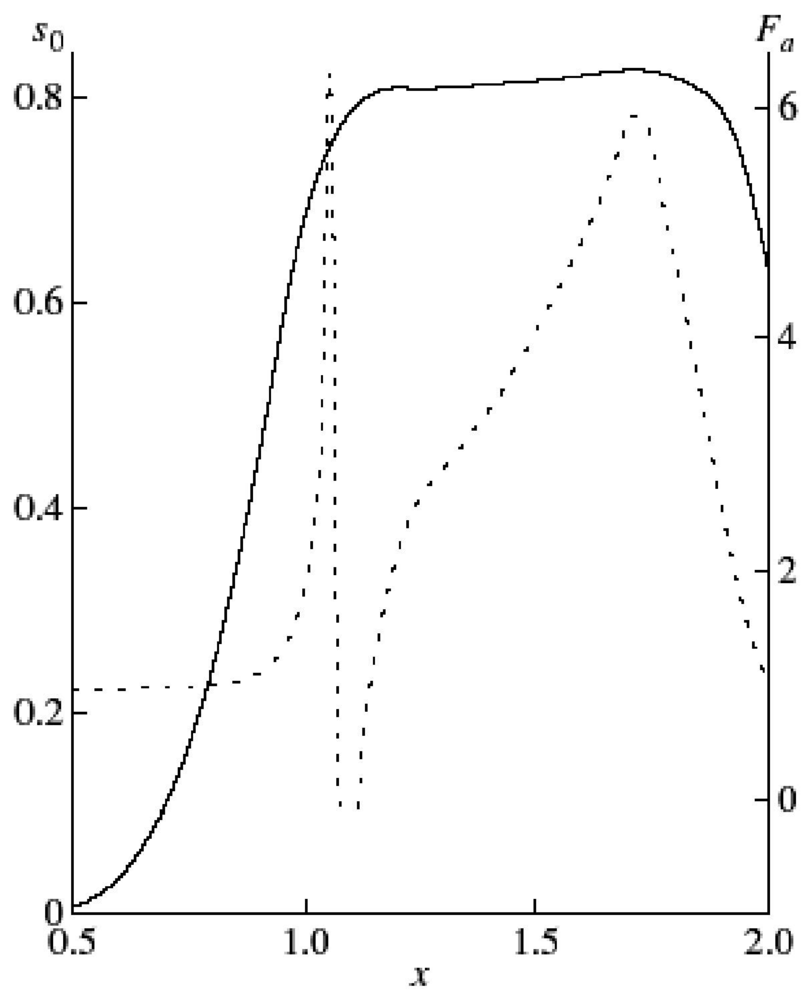


Fig. 7. Radial profiles of the entropy s_0 (solid curve) and anomaly factor F_a (dashed curve).

Kinetic Analysis of low-β Plasma

- Ideal MHD
  - Assumes adiabatic eq-of-state with $\gamma = 5/3$.
  - Ion FLR and $\eta_i \equiv (n_i \nabla T_i)/(T_i \nabla n_i)$ does not enter single fluid equations.

MHD $\rightarrow \frac{1}{p}\frac{dp}{d\psi} \leq 2\gamma\langle\kappa_\psi\rangle$ or $\hat{\omega}_{*p} \leq \gamma\hat{\omega}_d^{mhd}$

- There are several interesting orderings:
  - Ideal MHD (short mean free path, collisional)

    $\Omega_c > \nu > \omega_b > \omega_* \sim \omega_d \sim \omega$
  - Long mfp collisional

    $\Omega_c > \omega_b > \nu > \omega_* \sim \omega_d \sim \omega$
  - Collisionless Ions, Collisional Electrons ("Semi-collisional") expected in LDX

    $\Omega_{ce} > \omega_{be} > \nu_e > \omega_{*e} \sim \omega_{de} \sim \omega$

    $\Omega_{ci} > \omega_{bi} > \omega_{*i} \sim \omega_{di} \sim \omega > \nu_i$
  - Collisionless (expected in dipole reactor)

    $\Omega_c > \omega_b > \omega_* \sim \omega_d \sim \omega > \nu$

Kinetic Analysis

Kesner, Hastie, to be published in Phys Pl (2002).

- From DKE obtain $\tilde{f} = q\phi F_{0\epsilon} + J_0(k_\perp \rho)h$.

  with the non-adiabatic response, h, determined from:

$$\left(\omega - \omega_d + iv_{\|}\vec{b}\cdot\nabla'\right) h = -(\omega - \omega_*)q\phi F_{0\epsilon} J_0(k_\perp \rho) + iC(h)$$

  Assuming high bounce frequency the non-adiabatic response $h = h_0$ satisfies

$$(\omega - \overline{\omega}_d)\, h_0 = -\,(\omega - \omega_*)\, q\overline{\phi J_0} F_{0\epsilon} + i\overline{C}(h_0) \quad (1)$$

  with $\omega_* = \frac{\vec{b}\times\vec{k}_\perp\cdot\nabla' F_0}{m\Omega_c F_{0\epsilon}}$

$$\omega_d = \vec{k}_\perp \cdot \vec{b} \times \frac{(v_{\|}^2 \vec{b}\cdot\nabla\vec{b} + \mu\nabla B)}{\Omega_c},$$

$$\overline{\phi} = \left(\oint \frac{\phi(l)dl}{\sqrt{1-\lambda B}}\right)/\left(\oint \frac{dl}{\sqrt{1-\lambda B}}\right) \text{ and } \lambda = \epsilon/\mu.$$

- Dispersion relation: Solve for h_0, integrate over velocity space, apply quasi-neutrality.

Long mean-free-path Collisional Regime (Entropy mode)

- For $\nu_i, \nu_e \gg \omega, \omega_*, \omega_d$ obtain $\overline{C}(h_0) \approx 0$. Therefore

$$h_0 = \delta n \left(\frac{m/2\pi}{T+\delta T}\right)^{3/2} e^{-\epsilon/(T+\delta T)} \approx \left[\frac{\delta n}{n_0} + \frac{\delta T}{T}\left(\frac{\epsilon}{T} - \frac{3}{2}\right)\right] F_0$$

- Take the flux tube and velocity space average and assume the collision operator conserves particles and energy:

$$\int dl/B \int d^3v \overline{C}(h) = \int dl/B \int d^3v (\frac{\epsilon}{T} - \frac{3}{2}) \overline{C}(h) = 0$$

- We can now integrate Eq. [1] to solve for δn and δT in terms of "fluid" frequencies,

$$\hat{\omega}_{*j} = \frac{T\vec{k}_\perp \times \vec{b} \cdot \nabla n_0}{n_j m \Omega}$$

and $\hat{\omega}_d = \frac{cT(Rk_\perp)}{qV} \int \frac{dl}{B^2 R}(\kappa + \nabla B/B)$.

define $d \equiv \omega_{*pi}/\langle\omega_{di}\rangle \equiv \hat{\omega}_{*i}(1+\eta_i)/\langle\omega_{di}\rangle = -d \ln p / d \ln$

and $\overline{b} = \langle k_\perp^2 T_i / M_i \Omega_i^2 \rangle$

- For $k_\perp \rho_i \sim 0$ obtain at marginal stability [*Kesner, Phys Plasmas* **7** *(2000) 3837.*]

$$d = \frac{5}{7}\frac{1+\eta}{1-\frac{3}{7}\eta} \tag{2}$$

- Gyro-relaxation corrections: *Simakov, Catto, Hastie Phys Plasmas* **8**, *4414 (2001)*

  - $h_1 \sim O(\omega_*/\nu_{ii}) \rightarrow$ introduce “gyro-relaxation” corrections.

  - Mode shown to be flute-like.

Collisionless Ions - Collisional Electrons (Semi-Collisional) Regime

(Likely LDX regime)

Collisionless ion response: From Eq. 1

$$\frac{\delta n_i}{n_i} = -\frac{q_i \phi}{T_i} + \frac{q_i}{T_i} \int d^3v \frac{\omega - \hat{\omega}_{*i}(1+\eta_i(\epsilon/T_i - 3/2))}{\omega - \overline{\omega}_{di}(\epsilon,\lambda)} \overline{\phi} F_0$$

$$\equiv \frac{q_i}{T_i}(-\phi + \Lambda_i(\omega, \hat{\omega}_{*i}, \hat{\omega}_{di}))$$

- Consider particle motion in a point dipole field.
  - To obtain correct MHD response approximate

$$\overline{\omega}_{di}(\epsilon, \lambda) \approx \frac{2}{3}\frac{\epsilon}{T_i}\hat{\omega}_{di} \qquad (3).$$

- $\overline{\omega}_{di}(\epsilon, \lambda)$ Approximation

  We can better approximate
  $\overline{\omega}_{di}(\epsilon, \lambda) \approx \frac{2}{3}\frac{\epsilon}{T_i}\hat{\omega}_{di}\left(1 + \delta(\lambda B_{min} - 0.4)\right)$
  to obtain correction to $\overline{\omega}_{di}$. Find $\delta = 0.12$.

- This yields 1 % correction stability boundary.

Dispersion Relation - Semi-Collisional Regime

- Include collisional electron response and apply quasi neutrality:

  $$2\phi = \Lambda_i(\Omega, d, \eta) + \langle\phi\rangle\Lambda_e^c(\Omega, d, \eta) \quad (4)$$

  with $\Omega = \omega/\hat{\omega}_{de}$, $d = \hat{\omega}_{*e}(1+\eta)/\hat{\omega}_{de}$.

  Taking flux tube average yields: $2 = F_i + \Lambda_e^c$

  $F_i(\omega) = \int d^3v\ F_0\ \frac{\omega - \hat{\omega}_{*i}(1+\eta_i(\epsilon/T_i - 3/2))}{\omega - \frac{2}{3}\frac{\epsilon}{T}\hat{\omega}_{di}}$

- There is a flute eigenmode solution to Eq. 4.

- Dispersion relation can be written in form:

  $D(\omega) = \frac{d}{1+\eta}\left[F_1(\omega) + \eta F_2(\omega)\right] - F_3(\omega) = 0.$

- There is no marginal stability for $\omega/\hat{\omega}_{di} > 0$ and therefore no ion drift resonances. Thus have coincident real roots and at marginality $\partial D/\partial\omega = 0$.

  One can show $(F_2'F_3 - F_3'F_2) = -(3/2)(F_1'F_3 - F_3'F_1)$.

  Thus obtain

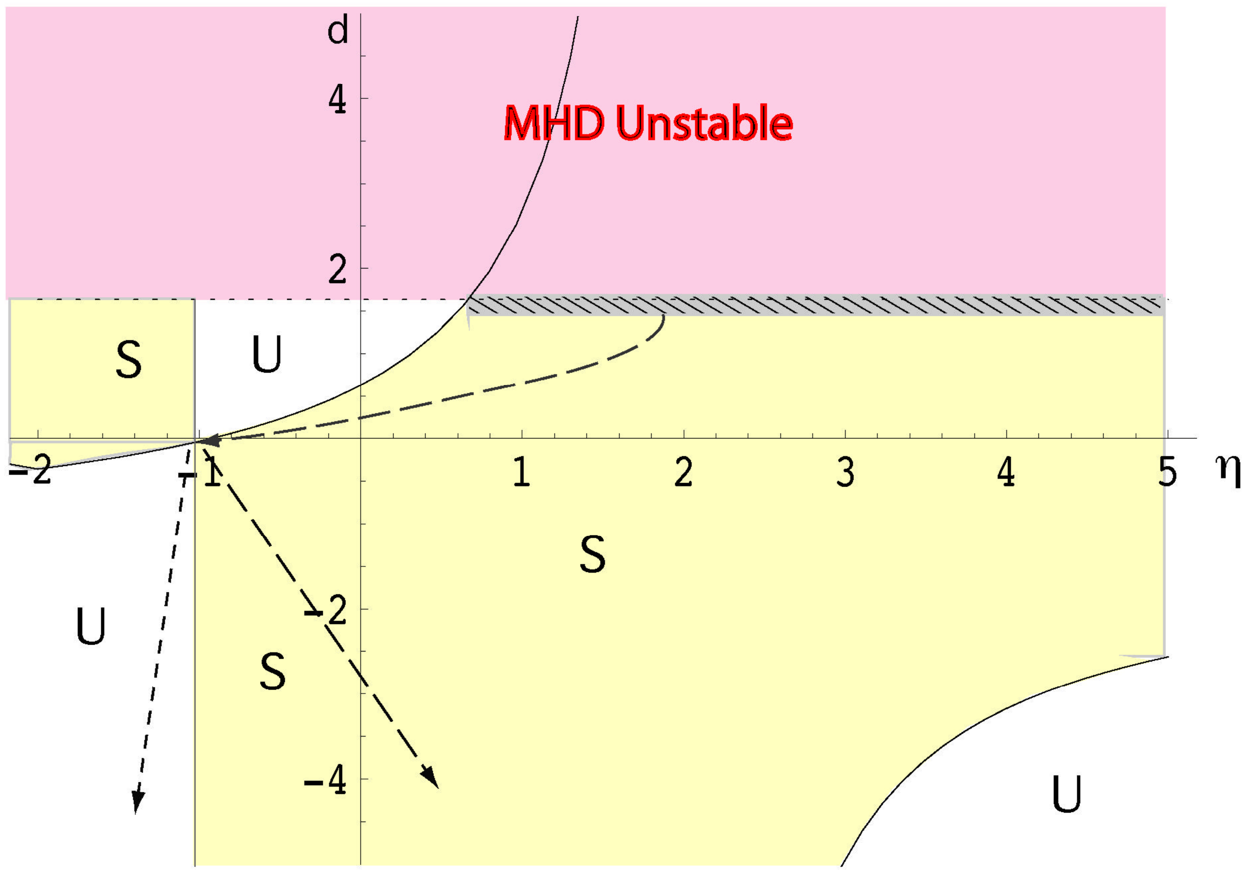
  $$(1 - \frac{3}{2}\eta)(F_1'F_3 - F_3'F_1) = 0$$

- Yields solution $\omega_{crit} = 0.32\ \hat{\omega}_{de}$ and

$$d = 0.66\ \frac{1+\eta}{1-0.51\ \eta}. \tag{5}$$

- Can evaluate stability numerically. Mathematica will evaluate error functions.
    - Nyquist plot indicates # of roots and stability.
    - Zero finder evaluates root.

Semi-collisional Mode (d=d ln p/d ln V)



Collisionless Ions and Electrons

- Rosenbluth [Rosenbluth, Phys. Fluids **11**, 869 (1968)] considered collisionless isothermal plasma ($\eta = 0$) in closed field line system. No FLR→ No MHD mode.
  - If any particles bounce in bad curvature always find an instability for $d > d_{crit}$.

    Note - In dipole all bounce in bad curvature.

- We consider arbitrary η and both good ($d < 0$) and bad ($d > 0$) curvature.

- Collisionless dispersion relation

$$2\phi = \int d^3v \frac{\omega - \omega_{*e}(1 + \eta_e(\frac{\epsilon}{T_e} - \frac{3}{2}))}{\omega - \frac{2}{3}\frac{\epsilon}{T}\hat{\omega}_{de}} \overline{\phi} F_{0e}$$

$$+ \int d^3v \frac{\omega + \omega_{*e}(1 + \eta_i(\frac{\epsilon}{T_i} - \frac{3}{2}))}{\omega + \frac{2}{3}\frac{\epsilon}{T}\hat{\omega}_{de}} \overline{\phi} F_{0i}$$

$$= \frac{1}{2}(\Lambda_e + \Lambda_{i0}) \int \frac{B d\lambda}{\sqrt{1 - \lambda B}} \overline{\phi} \qquad (6)$$

Taking the flux tube average can obtain $2 = \Lambda_e + \Lambda_{i0}$ Substitute into (6), take flux tube avg to obtain:

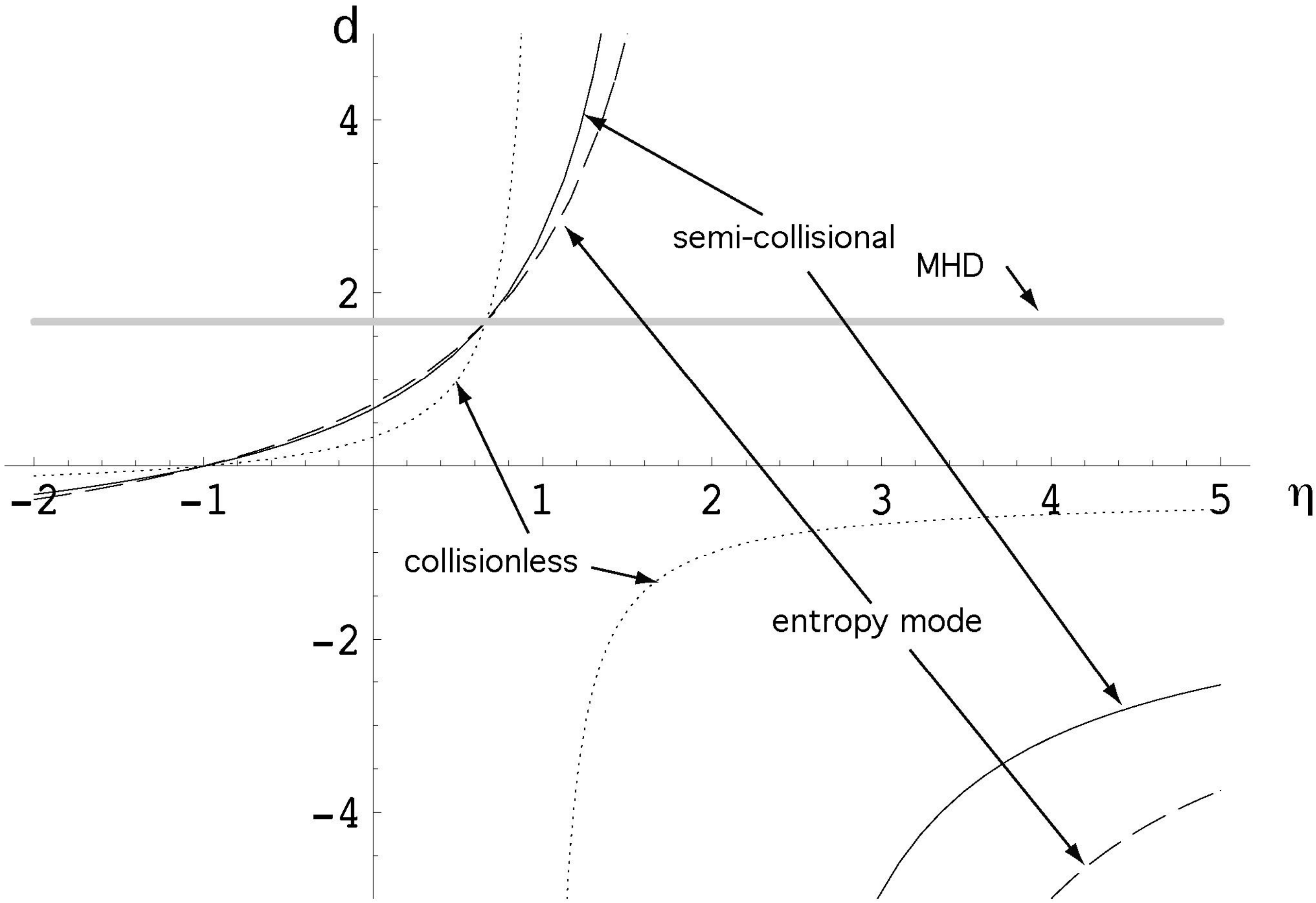
$$\oint \frac{d\ell}{B} \int \frac{B d\lambda}{\sqrt{1 - \lambda B}} (\phi^2 - \overline{\phi}^2) = \int \tau_b \, d\lambda (\overline{\phi^2} - (\overline{\phi})^2) = 0 \ .$$

Since $\overline{\phi^2} - \overline{\phi}^2 \geq 0$ obtain flute like, i.e. $\phi = \phi_0$ to order $k_\perp^2 \rho_i^2$.

- Following Rosenbluth look for marginality condition with Re[ω]= Im[ω]=0.

$$d = \frac{1}{3}\left[\frac{1+\eta}{1-\eta}\right]. \tag{6}$$

- Stability boundary is *similar* but *more restrictive* than collisional case.



d
4
2
-2
-4
-2
-1
1
2
3
4
5
η
semi-collisional
MHD
collisionless
entropy mode

Conclusions (Drift modes)

- 2 modes are present; MHD-like and drift mode
  - MHD mode stable when $d < 5/3$.
  - Drift mode driven by bad curvature ($d > 0$) and profile, i.e. η, effects.
- Collisionality is stabilizing; collisionless modes show larger area of instability.
- Levitated dipole
  - In region between the pressure peak and the wall $\nabla T < 0$, $\nabla n_e < 0$ and therefore $\eta > 0$.
  - At the pressure peak $d = 0$ and $\eta = -1$.
  - Between the pressure peak and the internal coil

    LDX: $\nabla T > 0$, $\nabla n_e > 0$ and $d < 0$, $\eta > 0$.

    Reactor: $\nabla T > 0$, $\nabla n_e < 0$ and $d < 0$, $\eta < -1$.

Summary of Dipole Theoretical Results

- *Between pressure peak and wall*

  MHD stable to interchange when $\delta(PV^{\gamma}) \geq 0$

  Stable to MHD ballooning when stable to interchange [3a, 4a]

  Stable to ES drift modes when stable to interchange for sufficient η [2a, 2b, 2c].

  ES "entropy" mode esentially unchanged in EM (high beta) region[2c]

  Unstable interchange modes evolve into convective cells [1d, 2d]

  Convective cells transport particles but not necessarily energy [1d, 2d].

  Convective cells can lead to non-local energy transport with H-mode-like edge [2d].

- *Between Internal Coil and pressure peak (good curvature region)*

  Can have drift modes when $\nabla(n_e) \leq 0$ [2b, 2c]

  Stable to all modes when $\nabla(n_e) > 0$ [2b, 2c]

can have Drift-cyclotron modes but little energy transport [2d]

Can have convective cells for non-uniform fueling [3d]

Implications for Dipole

- Levitated dipole is uniquely *simple* and *unorthodox* approach to plasma confinement.

  Inspired by magnetospheric physics observations. Naturally occurring high-β magnetic confinement.

  LDX is first experiment to directly test implications of stabilization by compressibility.

  Test the possibility of near-classical confinement below beta limit and non-local (convective) transport above limit.

  If predictions of high β and τ_E hold up may lead to advanced fuel (D-He3) fusion.

- Dipole area ripe for innovation:
- Coil set is simple; circular and non-interlocking coils.
- Challenging technology issues:
  - High TC superconducting coil within plasma.
  - Large vacuum chamber $\rightarrow$ low wall loading.

Poster will be available at the LDX web site: **http://www.psfc.mit.edu/ldx/**