Session BP1.114

Non-linear Simulations of the Levitated Dipole Experiment

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ABSTRACT

Theory predicts that the closed field line geometry of a levitated dipole configuration can support MHD stable plasmas at high beta values when the pressure gradient does not exceed a critical value. For sufficiently strong heating this stability limit would be violated and we expect instability to develop, leading non-linearly to the formation of convective cells. The NIMROD code [1] permits the study of the non-linear development of MHD modes, including resistive effects. NIMROD simulations have been performed for the LDX geometry. The simulations indicate the onset of MHD activity for sufficient heating power and for realistic (non ideal) heating profiles. After the onset of instability a toroidal mode number, n=1 mode is seen to dominate transport and to spread the plasma radially so that the pressure approaches the marginal profile. Later in time the amplitude of the modes decrease, the spectrum broadens (dominated by $10 < n < 20$) and the pressure profile approaches the marginal state.

[1] C.R. Sovinec and the Nimrod team, Journal of Computational Physics, Vol. 195, p. 355 (2004).

MHD Equilibrium and Stability of a Dipole

- MHD Equilibrium: Equilibrium is obtained for all β. At high β plasma expands in midplane.

    Free boundary equilibrium: Ref: Garnier, Kesner, Mauel, Phys Plasmas 6 (1999) 3431.

    Analytic equilibrium for point dipole: Krasheninnikov, Catto, Hazeltine, PRL **82** (1999) 2689.

- Ideal Stability:

    - Interchange stability when $\delta(pV^\gamma) > 0$ with $V = \oint d\ell/B$.

        Early Refs: Rosenbluth & Longmire, Ann Phys. **1** (1957) 120.

        Bernstein, Frieman, Kruskal, Kulsrud, Proc. R. Soc. London, Ser. A, **244** (1958) 17.

    - Ballooning modes

        Ballooning modes stable when interchange modes stable: Garnier et al, Ibid.

- Resistive MHD: Can have weak resistive mode $\gamma \propto \eta$, but the $\gamma \propto \eta^{1/3}$ mode is not present.

    Ref: Simakov, Catto, Ramos, Hastie, Phy Pl **9** (2002) 4985.

Heating Experiments in LDX

- 3 KW ECRH at 2.45 and at 6.4 GHz

  - Typical Shot 40917012: Observe the following;

    During first 200 ms the plasma is quiet. Diamagnetism and x-ray intensity rises.

    After t=200 ms

  * Noise on Mirnov coil (magnetic pickup coil) becomes significant

  * Diamagnetic signal changes slope.

  * Edge Langmuir ion saturation current goes from negative to positive.

  * Diamagnetism drop (β collapse ?) observed later in time (t$\sim$1.5 s) and at high β.

- **Possible explanation:**

  - Plasma pressure rises in footprint of heating source forming small plasma in the vicinity of the coil. The negative signal on the Langmuir probe indicates some hot electrons reaching the probe.

    After t=200 ms turbulence observed on Mirnov loops and plasma spreading indicated by Langmuir probes.

A burst of neutral gas is released when the plasma scrapes off and the hot electrons pitch angle scatter on neutrals and are lost.

- Note: LDX operated in supported mode with strong ECRH is not a MHD plasma

  Loss cone, due to supports, means pitch-angle-scatter is an important loss mechanism.

 Kinetic terms due to hot electrons complicate the stability boundary.

This general progression of events,

- **a quiet period followed by**

- **a rapid expansion of the plasma due to turbulence**

- **continued rise of diamagnetism with continued fluctuation activity**

is observed in NIMROD simulations.

  NIMROD not run long enough in time to observe high β instability, i.e. β collapse.
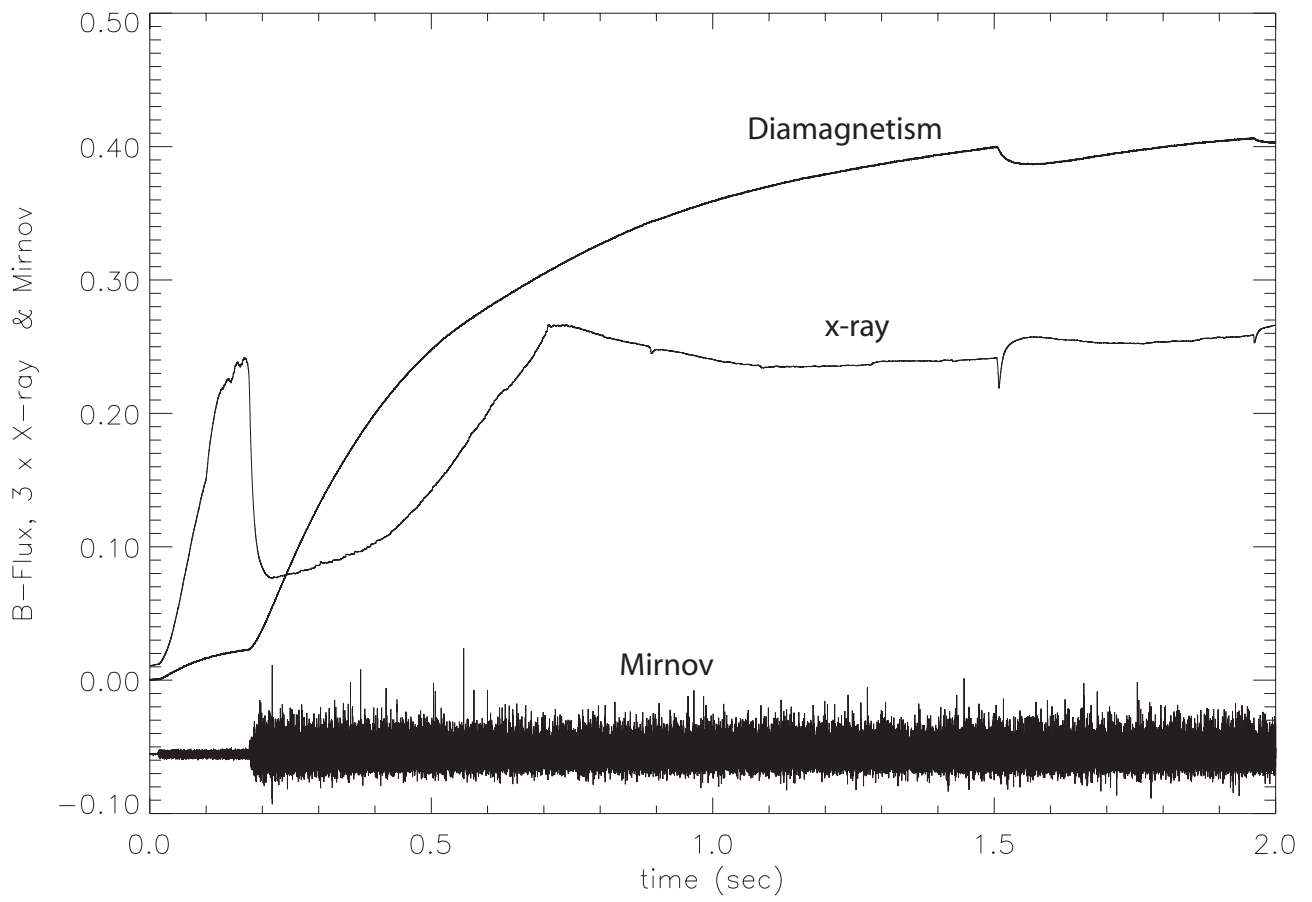
NIMROD Code

- Solves non-linear resistive MHD equations

- Initial value calculation in real geometry

    Provide realistic heating and particle source

    Approximate $\chi_\perp = constant$. Use higher order (p=3) elements to permit $\chi_\parallel \gg \chi_\perp$.

    Kinematic viscosity approximation in momentum equation. No diamagnetic terms.

- NIMROD equations

$$\frac{\partial \mathbf{B}}{\partial \mathbf{t}} = -\nabla \times \mathbf{E} + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$

$$\mathbf{E} = -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) = \nabla \cdot D\nabla n$$

$$m_i n (\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}) = \mathbf{J} \times \mathbf{B} - \nabla p + m_i \nabla \cdot \nu n \nabla \mathbf{U}$$

$$\frac{n}{\gamma - 1}\left(\frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T\right) = -\frac{p}{2}\nabla \cdot \mathbf{U} - \nabla \cdot \mathbf{q} + \mathbf{Q}$$

$\mathbf{U}$ = flow velocity, Q the heat source density, $\mathbf{q}$ the heat flux:

$$\mathbf{q} = -n\left[\chi_\| \hat{\mathbf{b}}\hat{\mathbf{b}} + \chi_\perp(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})\right] \cdot \nabla T$$

Boundary Conditions: Dirichlet for $\mathbf{B}_\perp$, T, $\mathbf{U_i}$. Flux for n, $(F_n = -D\nabla n)$.

- ECRF heated plasma $\rightarrow$ hot electron component

  - If $p_{eh} \propto p_{core}$ stability of hot electron interchange mode requires $\delta(p_{eh}V^{\gamma}) > 0$, similarly to MHD.
    Poster **BP1.141**, Krasheninnikova and Catto.

- Non-Linear MHD in hard-core pinch geometry (high aspect ratio approx):

  - Convective cells will develop when interchange limit is exceeded. *Leads to particles transport but not necessarily energy transport.*

    Ref: Reduced MHD: Pastukhov and Chudin, Plasma Physics Report, **27**, (2001) 963.

    PIC simulation: Tonge, Leboeuf, Huang, Dawson, Phys Pl **10** (2003) 3475.

- **We want to simulate MHD instability in non-linear regime in real 3-D dipole geometry**

  We consider configuration similar to LDX with strong localized heating so as to drive instability

In ideal case dipole can heat up to maximum pressure determined by $pV^\gamma = constant$ profile.

- DIPEQ code produces Grad-Shafranov equilibrium for $pV^\gamma = constant$. For $\beta_{max} = 36\%$ equilibrium for LDX paramters are:

Dipole (LDX) Parameters

Floating coil major radius	0.35 m
Coil Current	1.5×10^6 A
Vacuum vessel midplane radius	2.5 m
Peak field near coil	5 T
Edge pressure	1.3 Pa
Peak pressure	150 Pa
Midplane radius of $R(p_{max})$	1.16 m
Peak β	0.36

Psi and |B| Contours
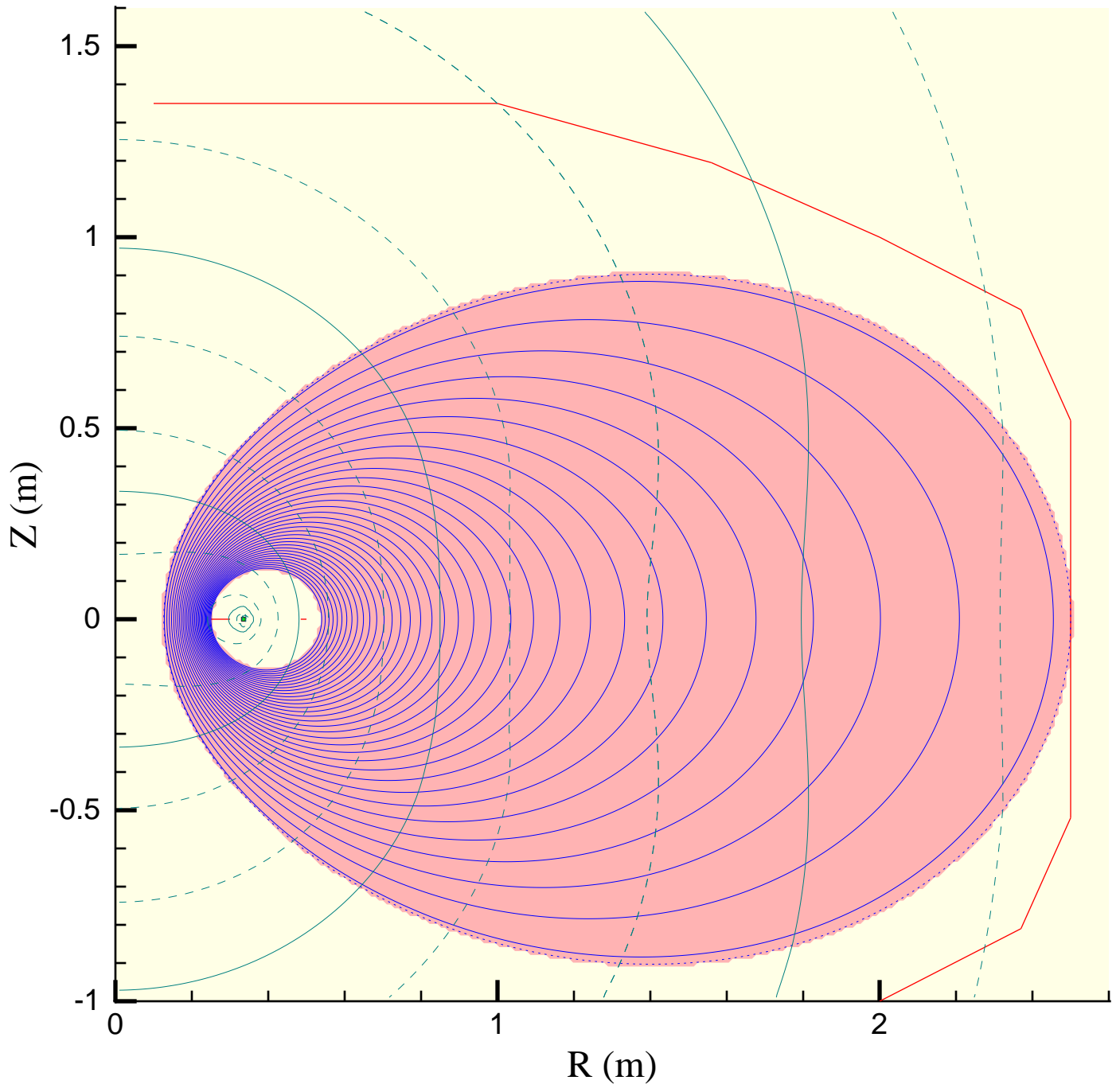
Nimrod Input Plasma Parameters

Edge Temperature		0.4 eV
Perp thermal diffusivity ($\chi_\perp$)		$1\ m^2/s$
Parallel thermal diffusivity ($\chi_\parallel$)		$10^5\ m^2/s$
Perpdiffusion coefficient(D)		$400\ m^2/s$
Resistivity/μ_0		$10\ m^2/s$
Kinematic viscosity		10 - $1000\ m^2/s$
Alfven growth rate		0.1-$1.5 \times 10^6\ s^{-1}$
Heating function **:	Power	120 KW
	$R_0(Q_{Max})$	1.15 m
	FWHM	0.32 m

Other Nimrod Input Parameters

Poloidal grid	30 x 60
Number of toroidal modes	42
Finite element basis function degree	3 †

** High power used to speed up calculation

† High order finite element necessary to eliminate $\chi_\parallel$ cross-field heat leakage in high β system

Stages of Discharge Observed in Simulation

NIMROD allows us to view growth and interaction of the unstable spectrum of modes.

- $0 < t < 125$ μs (depending on heating rate). Stable heat up

    Pressure increases without observable losses

- $125 < t < 400$ μs. Linear instability growth:

    Higher-n modes grow fastest

    Pressure increases without observable losses

- $t > 400$ μs. Non-linear stage:

    Modes saturation at macroscopic levels.

- $400 < t < 600$ μs. n=1 mode dominant.

    Convective cells present. Pressure profile broadens to fill volume.

    Not steady state but convection pattern persists

- $0.6 < t < 5$ ms. Spectrum of modes dominated by $10 < n < 20$. These are possibly resistive modes.

    - Pressure continues to evolve toward $pV^{\gamma} = constant$ profile.

- Edge pedestal develops due to viscosity and velocity boundary condition ($v_{edge} = 0$).

* **Recent calculation out to t=5 ms (31,000 time steps) is approaching the critical pressure profile.**

  The n=1 mode appears to be most efficient mode for large scale transport and may appear when $p \propto 1/V^\gamma$ throughout profile. (Calculated profile has not yet reached this point).
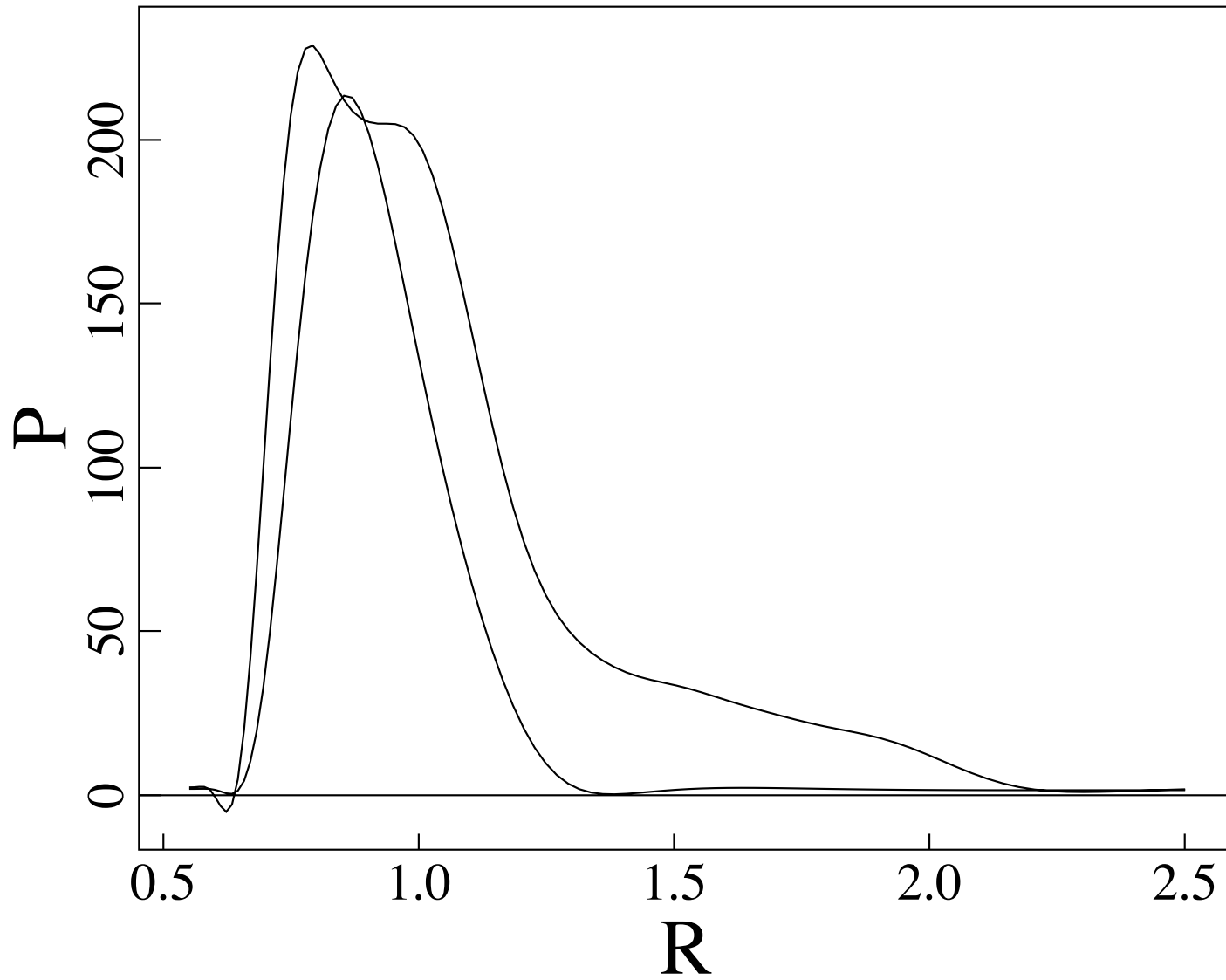
Pressure contours in midplane in turbulent plasma at t=0.51 (n=1 dominant) and t=0.64 ms (broadband)
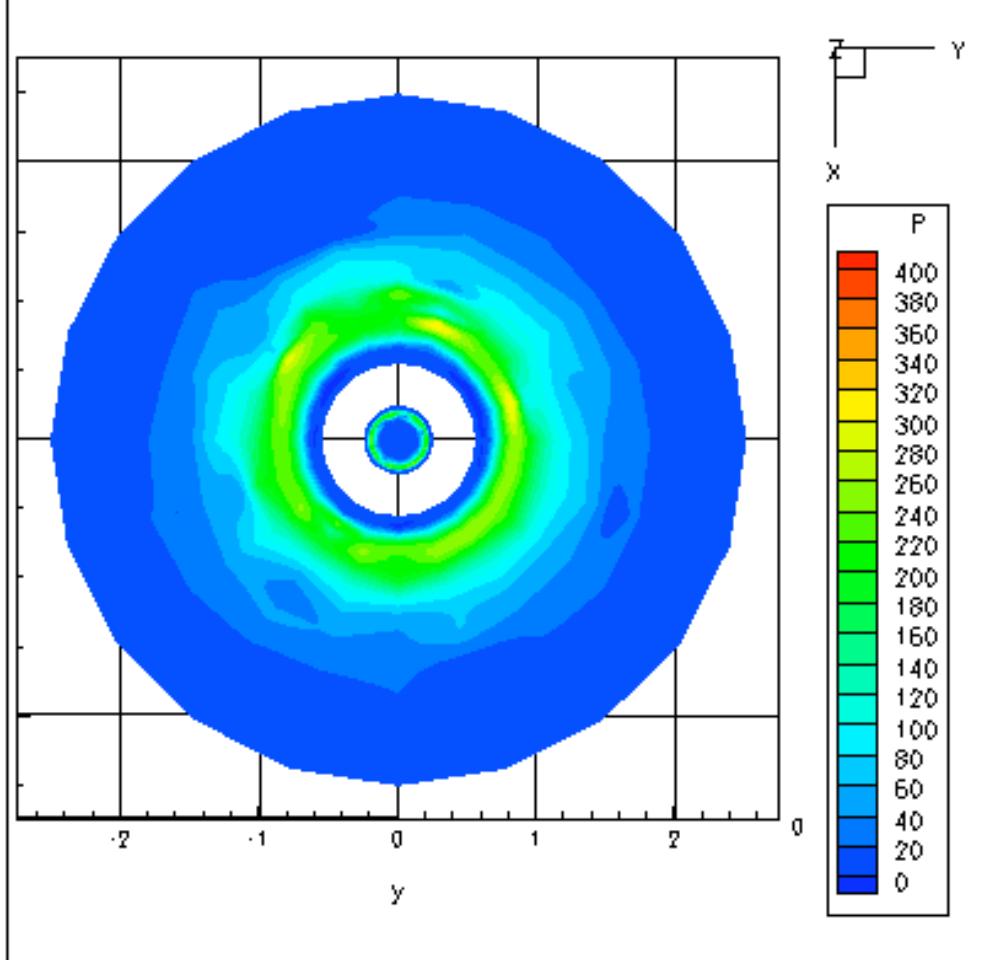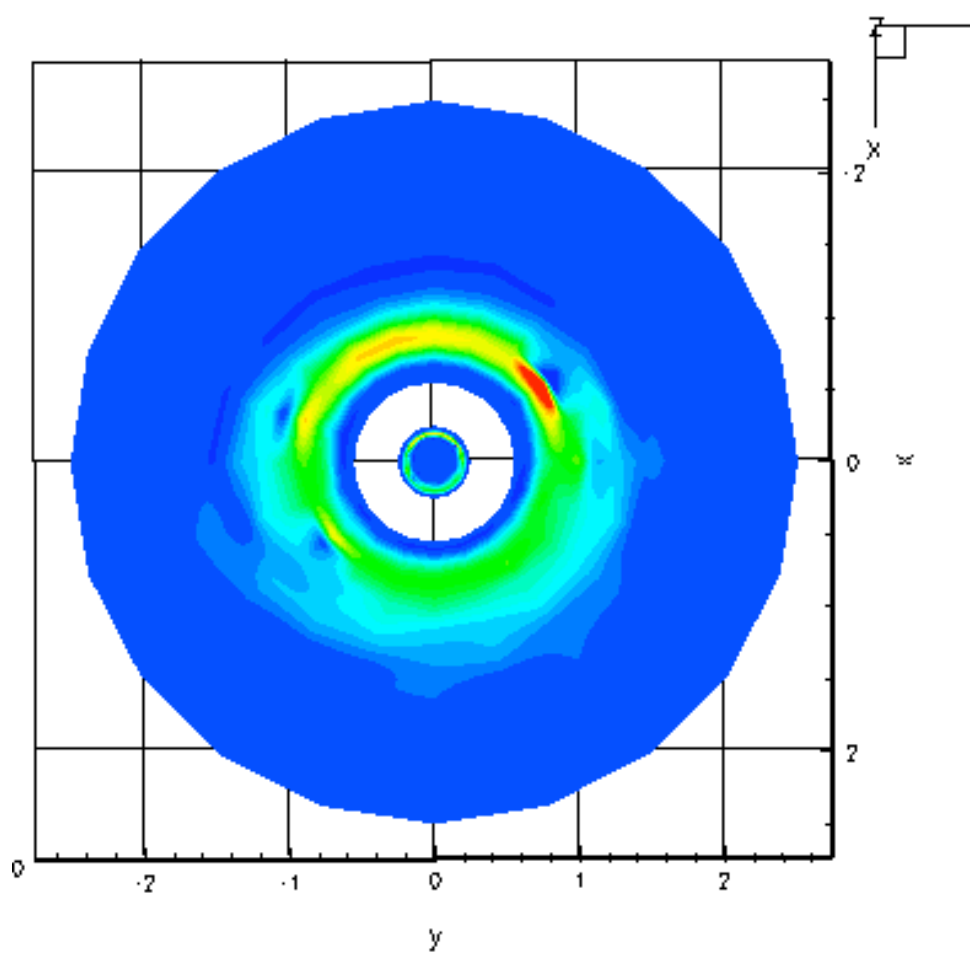
Midplane pressure profile before and after instability

The n=1 mode causes the pressure profile to broaden ($\phi = 0$ shown).

Re P Along Slice

Amplitude of modes between $0 < t < 0.6 \ ms$

$log(E_k)$ vs t for $0 < k < 42$.
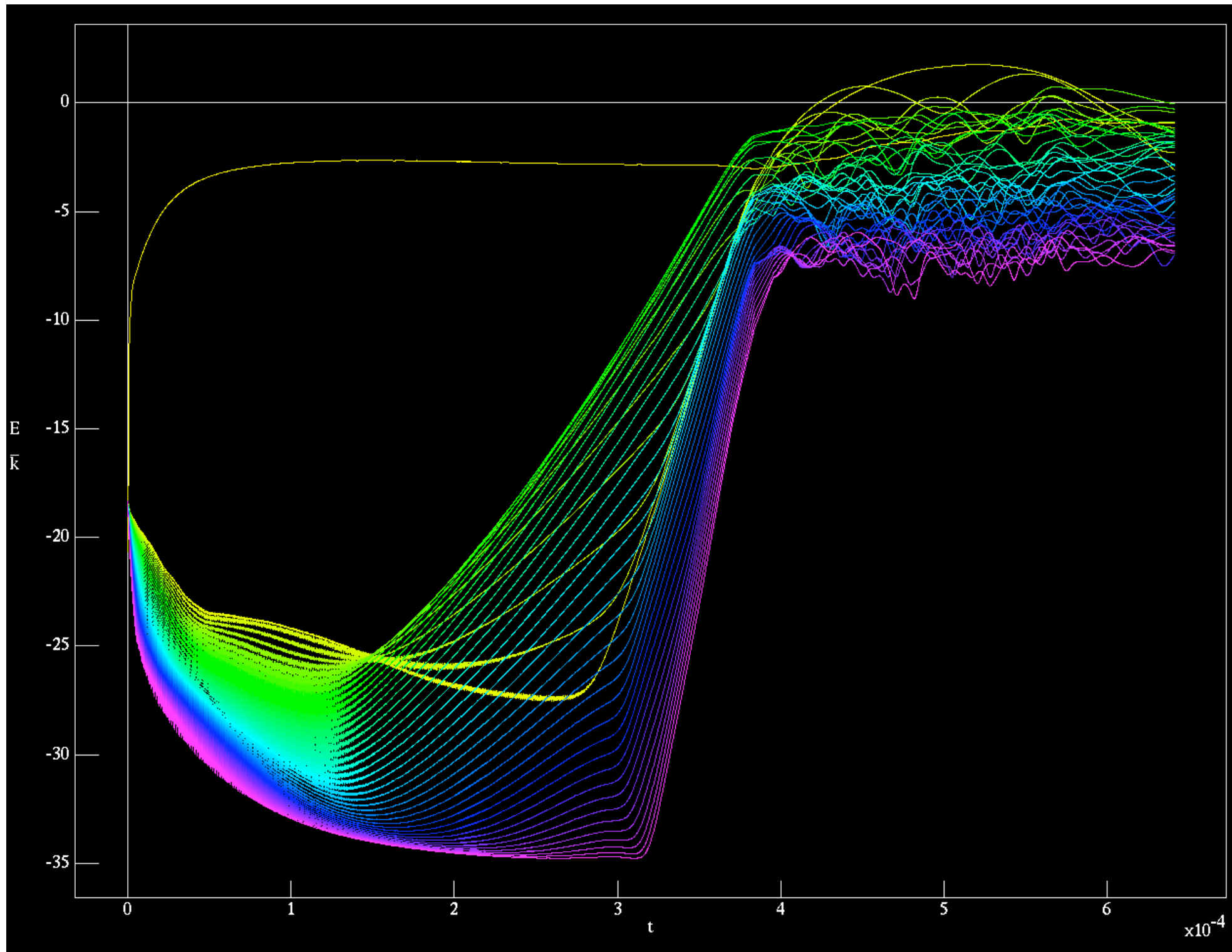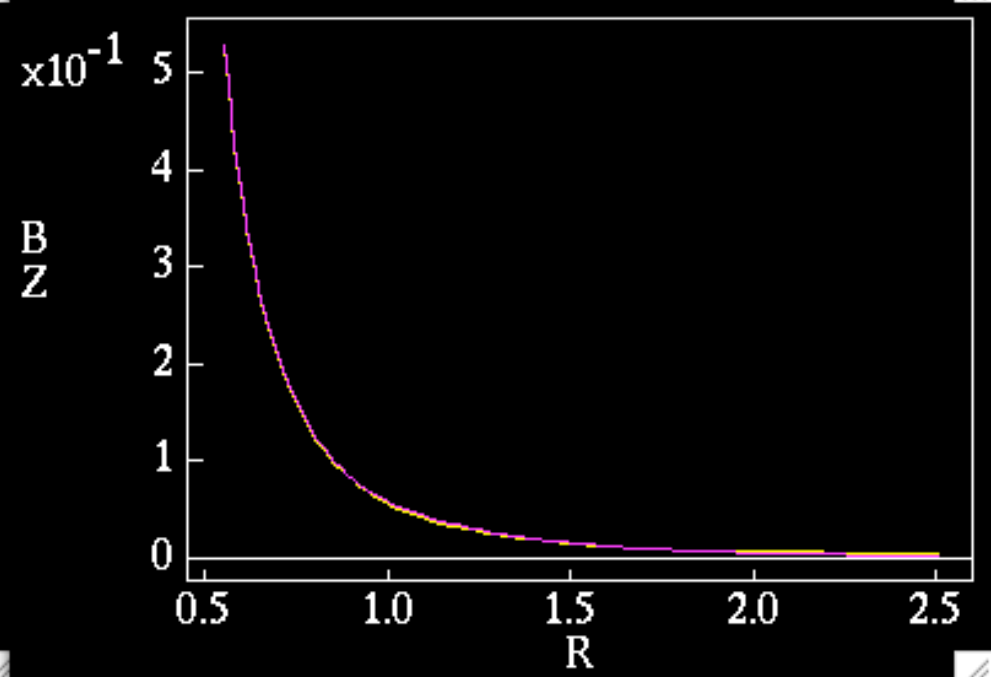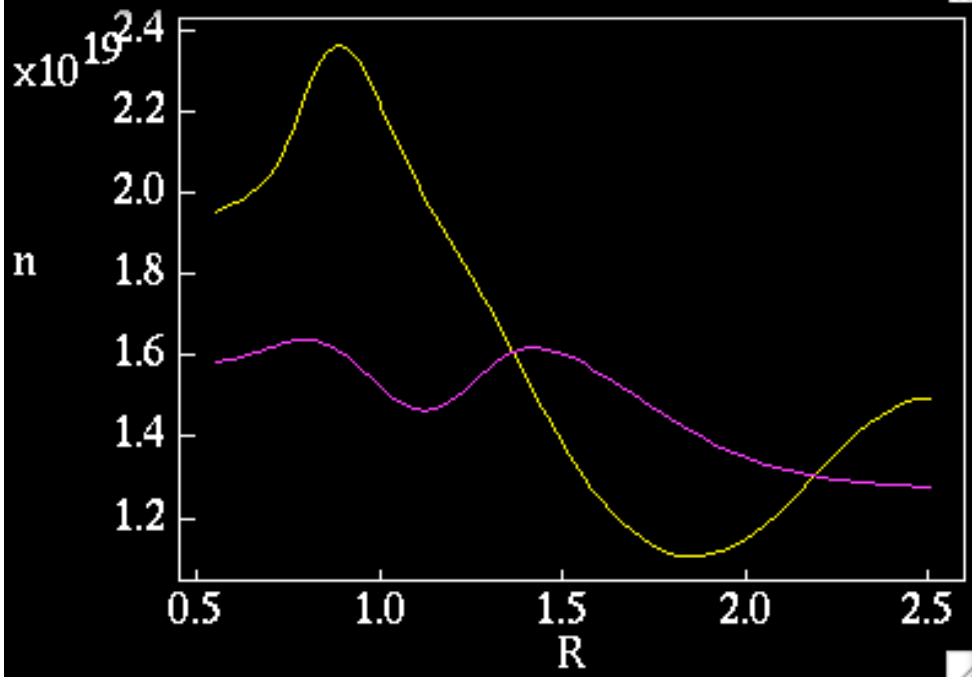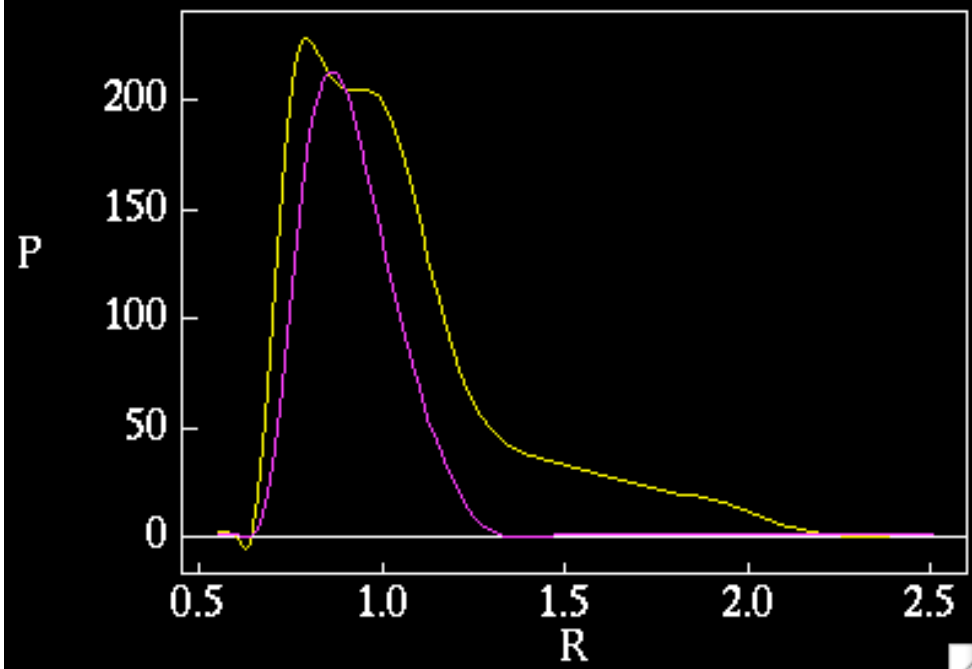
Spectra of modes dominated by n=1.
E_k at t=0.5 ms

Kinetic Energy vs. k

E_k

k
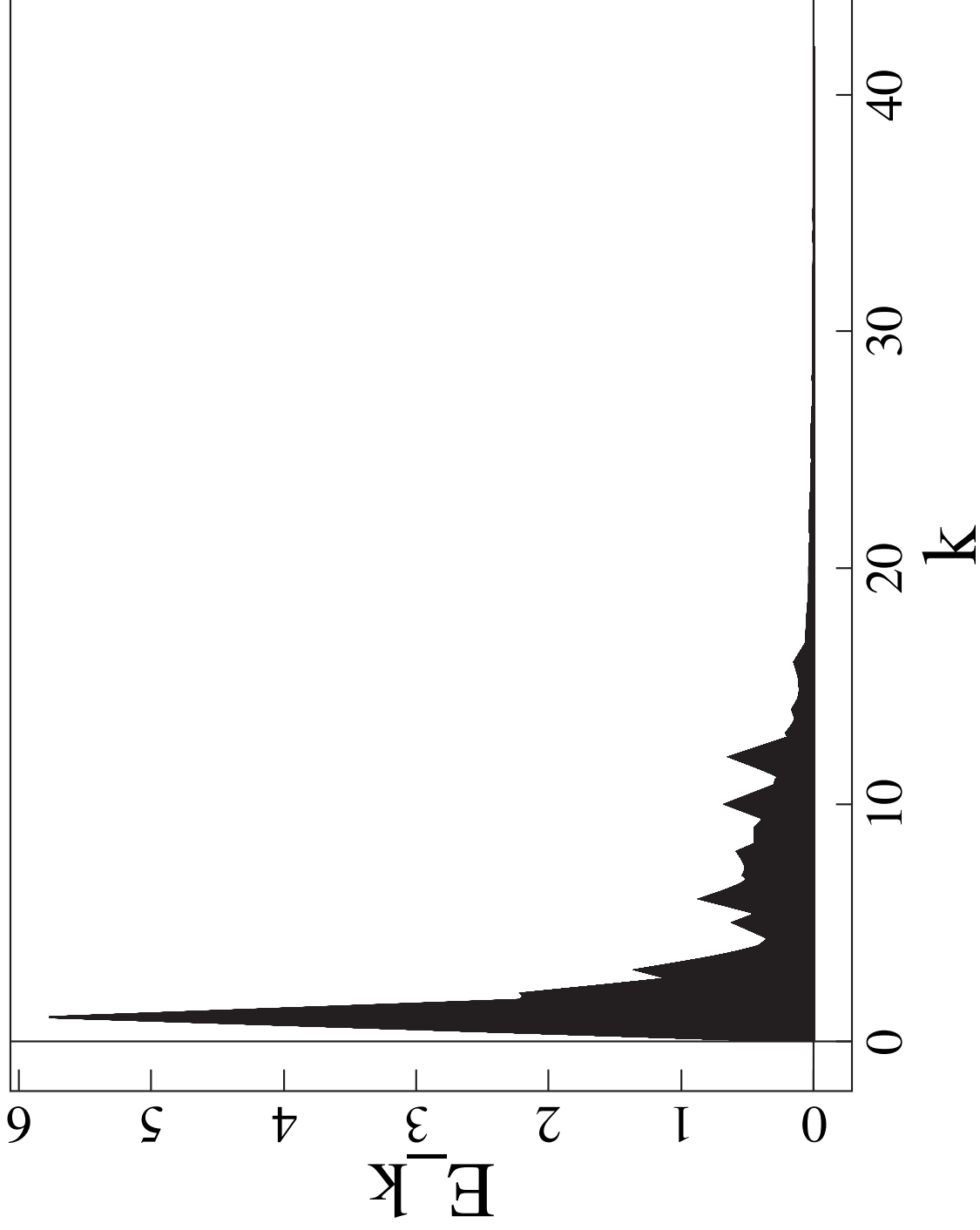
t=0.5 ms

Heat up and turbulent expansion $0 < t < 0.6\ ms$

Parameters at R=0.85 m for $0 < t < 0.6 \; ms$
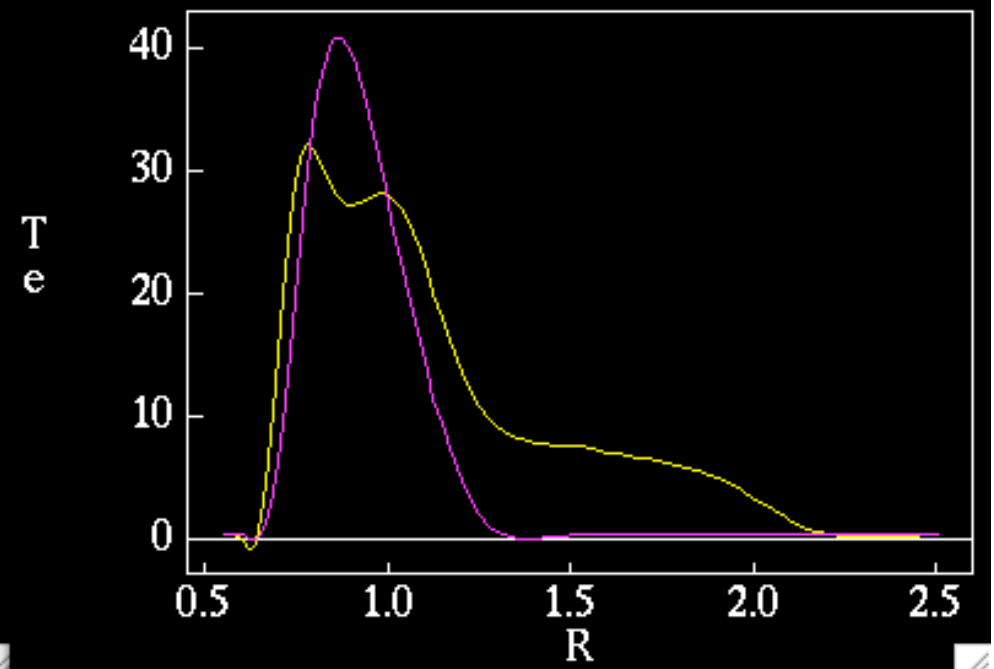
- $0 < t < 0.125$ ms: p(ψ) rises linearly w/o instability

- $0.125 < t < 0.4$ ms: p(ψ) rises linearly w/o instability

- $0.4 < t < 0.6$ ms: n=1 mode dominates as pressure profile spreads out to fill vacuum chamber.

- $0.6 < t < 5$ ms: non-linear regime. Peak pressure grows, pedestal appears, turbulence level provides energy transport.
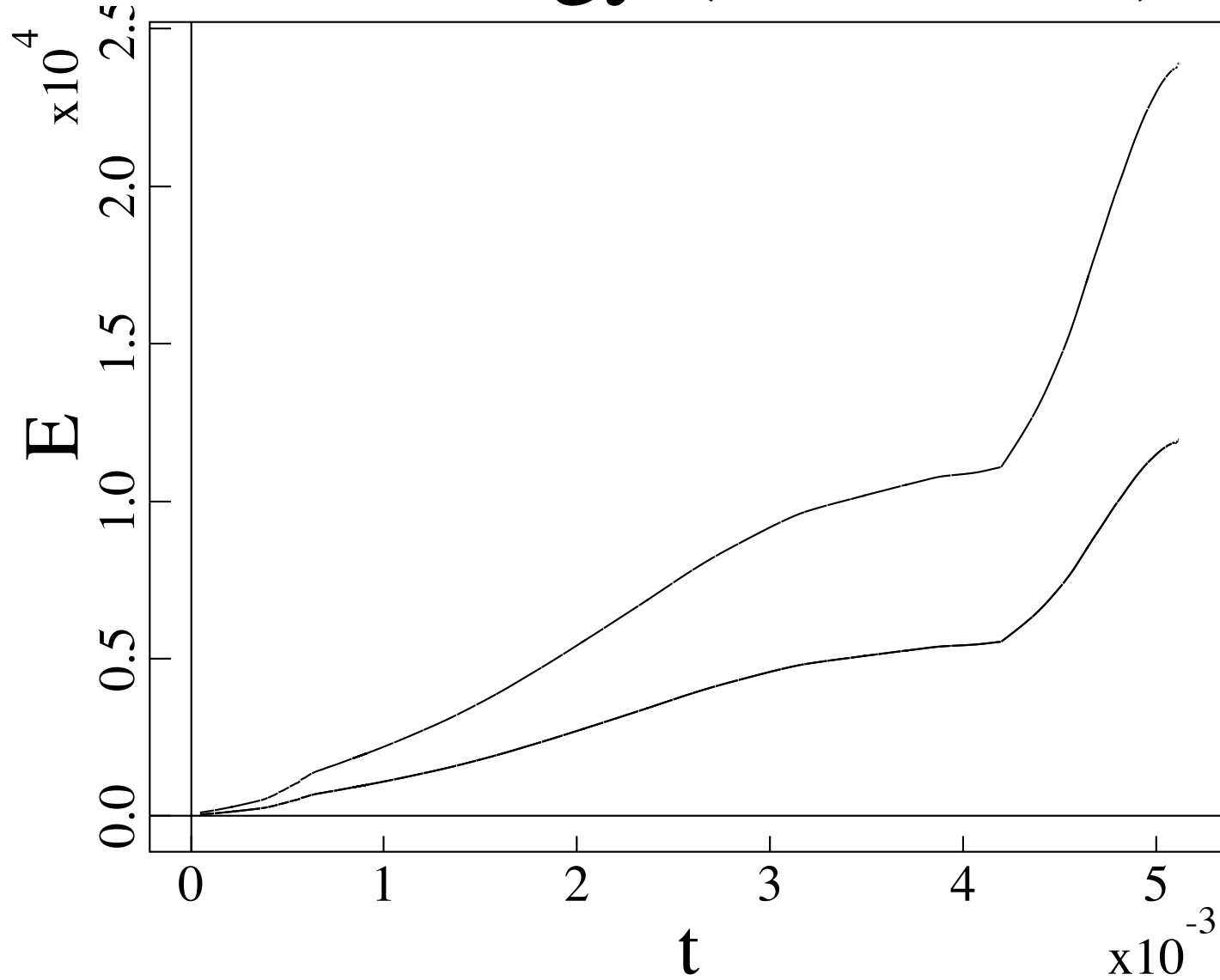
- $t > 0.6$ ms ?

Amplitude of modes between $0 < t < 5\ ms$

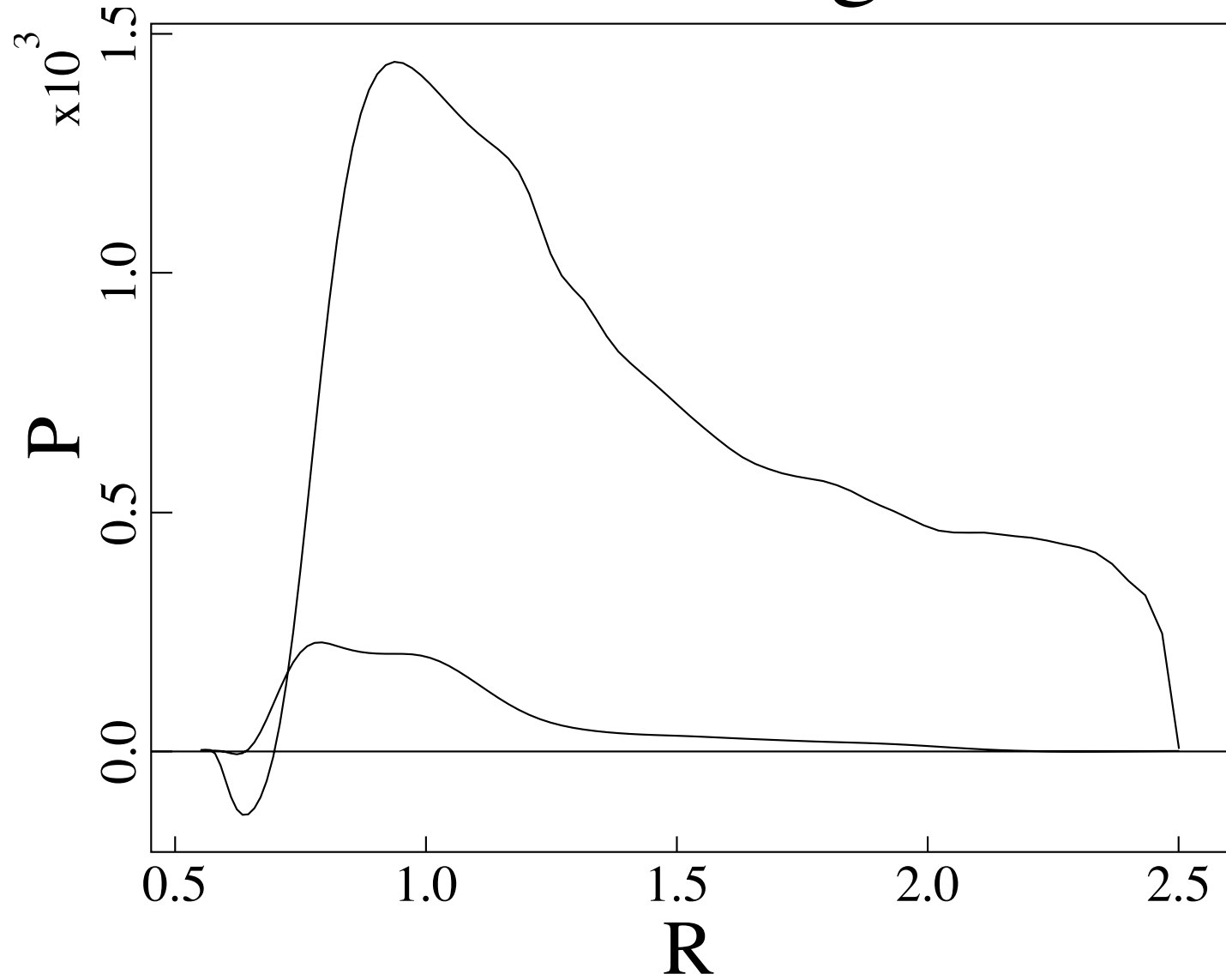$log(E_k)$ vs t for $0 < k < 42$.
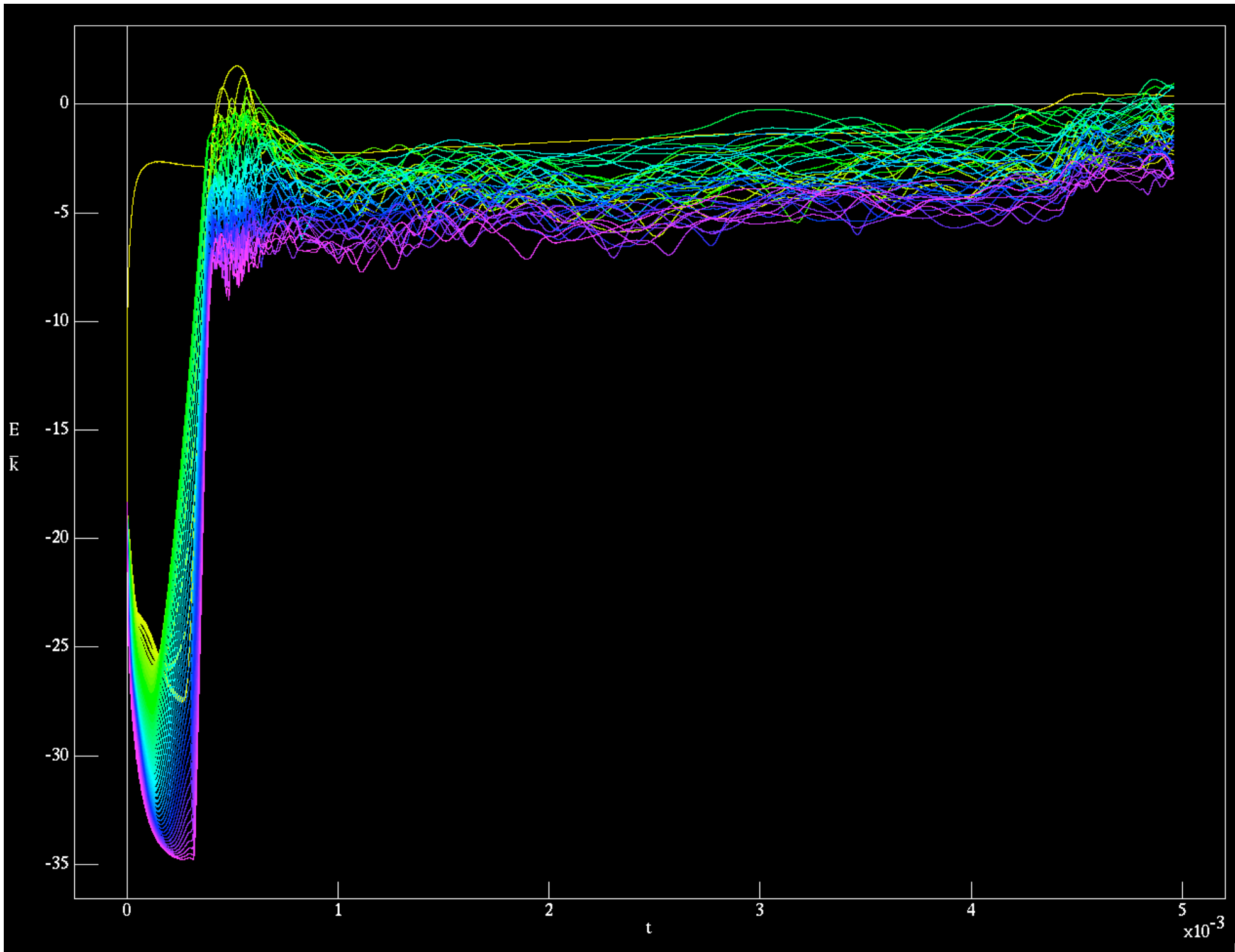
Plasma stored energy (J) vs time

Internal Energy (tot, el, ion) vs. t

Midplane pressure profile after heat up phase
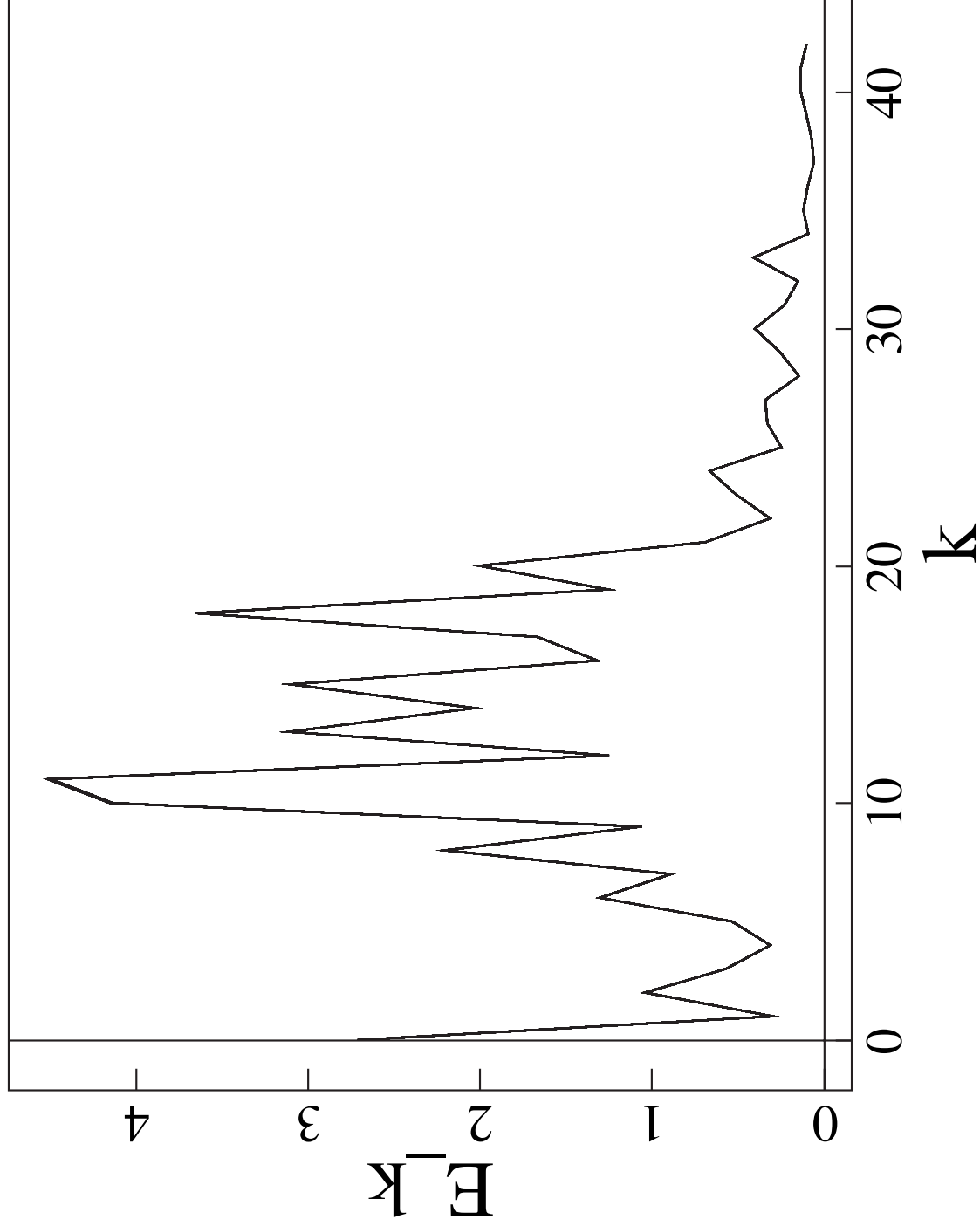
- Most of profile at $pV^\gamma = const$ limit

- Since viscosity eliminates flows near the edge a pedestal forms at $\phi = 0$ with sufficient slope for $\chi_\perp \nabla T$ to transfer the power across the boundary.

Re P Along R

$x10^3$

P

R

Broad spectra of modes dominated by $10 < n < 20$. as pressure rises for $t > 0.5$ ms.

Kinetic Energy vs. k

E_k

k

t=5.1 ms

CONCLUSIONS

- Strong heating with narrow heating profile can drive pressure profile to the MHD stability limit.

- MHD instability will develop and grow up in $\sim$ 100 μs
  - Higher modes ($n > 20$) will grow fastest in the linear stage

  - In non-linear stage n=1 will transport plasma to fill available volume.

- Broadened pressure profile is maintained by $10 < n < 20$ resistive modes.

  - Pedestal observed at edge caused by viscous damping of convective flows.

- **Calculation not yet complete.** MHD is expected to impose a stiff pressure profile close to the marginal profile, $pV^{\gamma} = constant$.

GENERAL COMMENTS

- Non-linear MHD offers a *guess* for the behavior of a strongly heated dipole.

    - Result is dependent on boundary conditions, particularly assumption $\vec{v}_{edge}=0$.

    - Heating profile together with $\chi_\perp(\vec{r})$ determines pressure profile and therefore stability boundary (calculation assumes $\chi_\perp = 1$).

      $\chi_\perp(\vec{r})$ will be determined by micro turbulence.