ABSTRACT

Magnetic Diagnostics in LDX

Magnetic diagnostics will play an essential role in understanding the equilibrium and stability of LDX plasmas. Flux loops, poloidal field coils, Hall probes, and Mirnov coils have been installed and tested for the first experimental campaign. These measurements will provide the boundary field and flux values needed as inputs to a Grad-Shafranov solver for equilibrium reconstruction. Specifically, the boundary magnetic signals constrain the location and shape of the pressure function, p(□). The sensors, excluding the Mirnov coils, have been installed in accordance with an optimization scheme that maximizes their sensitivity to diamagnetic currents. The Mirnov coils have been designed to detect plasma fluctuations up to MHz range to characterize MHD activity. Initial testing and calibration of the sensors have been performed *in-laboratorio* using a medium-sized Helmholtz pair, and the results have subsequently been compared to data obtained *in-loco* with a large Helmholtz pair installed on the LDX vacuum chamber. Additionally, an equilibrium reconstruction model that incorporates pressure models more appropriate for the LDX plasmas than the more common polynomial expansion has been completed.

WHAT MAGNETIC DIAGNOSTICS WILL ACCOMPLISH

- Deduce the current and pressure profiles via the reconstruction scheme
- Obtain information regarding plasma shape and position
- Extract plasma beta profile
- In conjunction with density diagnostics, construct the temperature profile to cross-check with that obtained from x-ray diagnostics
- Get the total stored (kinetic + magnetic) energy density

MAGNETIC DIAGNOSTICS OVERVIEW FOR THE LDX

- 18 poloidal B-coils to measure orthogonal fields at 9 different locations
- 9 poloidal flux-loops
- 18 Hall probes directly mounted on the Bp-coils to supplement the coil measurements during long pulse shots
- 8 toroidally distributed Mirnov coils to measure fast plasma fluctuations

EQUILIBRIUM RECONSTRUCTION ITERATION MECHANISM

Iterative Grad-Shafranov Equation is used along with the pressure model:

$$\square^{\square}\square^{k+1} = -\square_0 R J_{\square}^{k}(R,\square^{k},\square_n^{k})$$

$$J_{\square}^{k} = RP'(\square^{k},\square_n^{k})$$

$$P'(\square^{k},\square_n^{k}) = \square_{n=1}^{n_n}\square_n^{k}\,\square_n(\square^{k})$$

In addition, the $\square_n^k$'s are found in such way as to minimize the merit function $\square$:

$$\square_n^{k+1} = [L^{\dagger}]_{nj}^{k}(M_j - C_j^{k}) + \square_n^{k}$$

- L.L. Lao, et al., Nucl. Fusion **30**, 1035 (1990).

THE MERIT FUNCTION AND ITS ROLE IN THE ITERATION SCHEME

The merit function χ^2 is defined as following:

$$\chi^2 = \sum_{j=1}^{n_m} \frac{(M_j - C_j^k)^2}{\sigma_j^2} \quad \text{where}$$

M_j	=	measurement value at the j^{th} detector
C_j^k	=	calculated value from $\square^k$ or J^k at the j^{th} detector position
σ_j	=	measurement error

Since $C_j^k = C_j^k(\square^k, \square_n^k)$ we can choose $\square_n^k$ in each iteration so as to minimize χ^2

PROSPECTIVE PRESSURE MODELS

One of the simplest and most commonly used pressure models is the polynomial:

$$\square_n(\square) = \square^n \qquad \text{where } \square \equiv \frac{\square - \square_{FCFS}}{\square_{LCFS} - \square_{FCFS}} \quad \text{(Normalized Flux)}$$

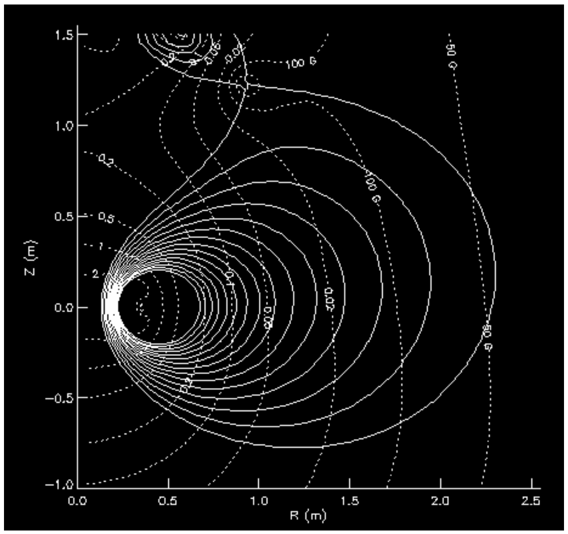
$$\boxed{P'(\square,\square_n) = \square_{n=1}^{n_p} \square_n \square^n}$$

In LDX, the adiabatic compressibility condition allows for a potentially better model of the following form:

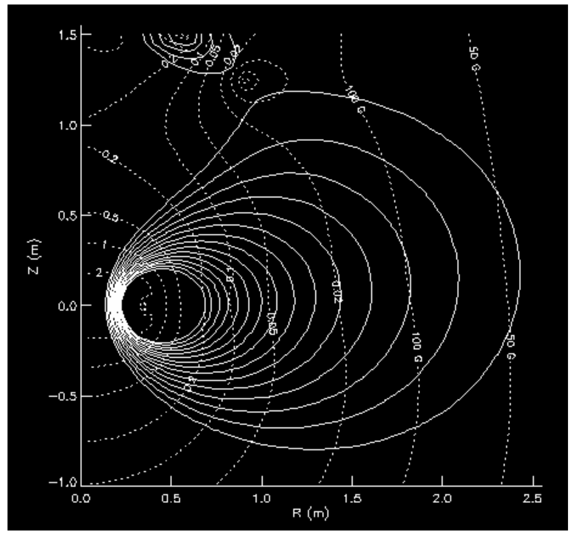
$$P(\square,\square_p,P_{edge},g) = \begin{cases} P_{edge}[V(\square)/V_{edge}]^{-g} & \text{for } \square > \square_p \\ P_{edge}[V(\square)/V_{edge}]^{-g} \sin^2[(\square/2)(\square/\square_p)^2] & \text{for } \square < \square_p \end{cases}$$

- D.T. Garnier, et al., Phys. of Plasmas 6, 3431 (1999).

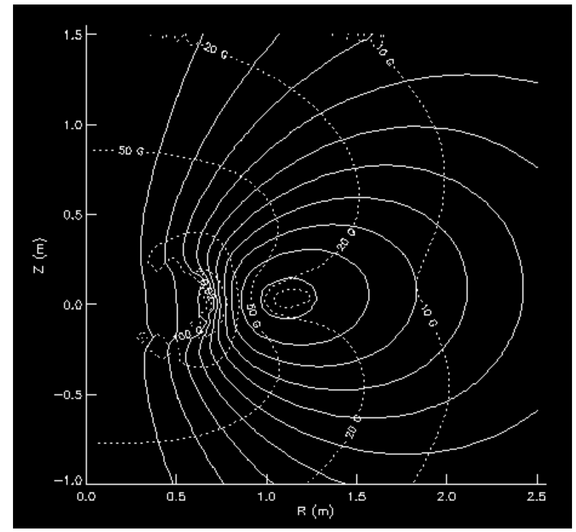
TYPICAL FIELD CONFIGURATIONS



Vacuum Field



Low Beta Plasma Field



Difference Field

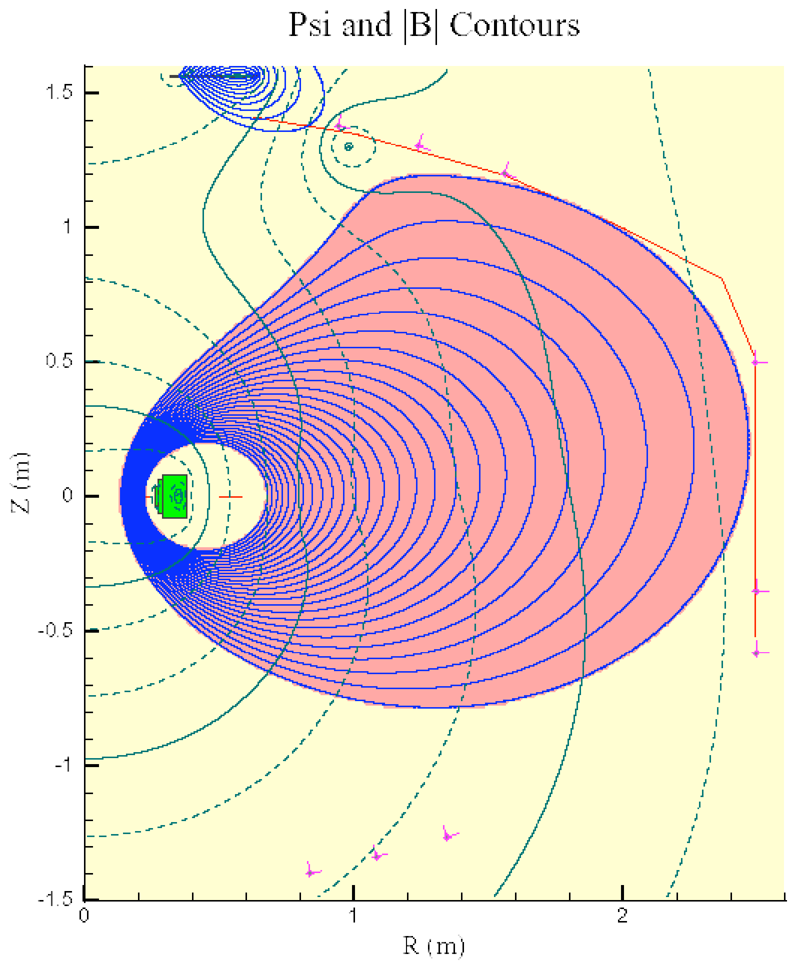
Assumed Conditions:

$P_{edge} = 1Pa$

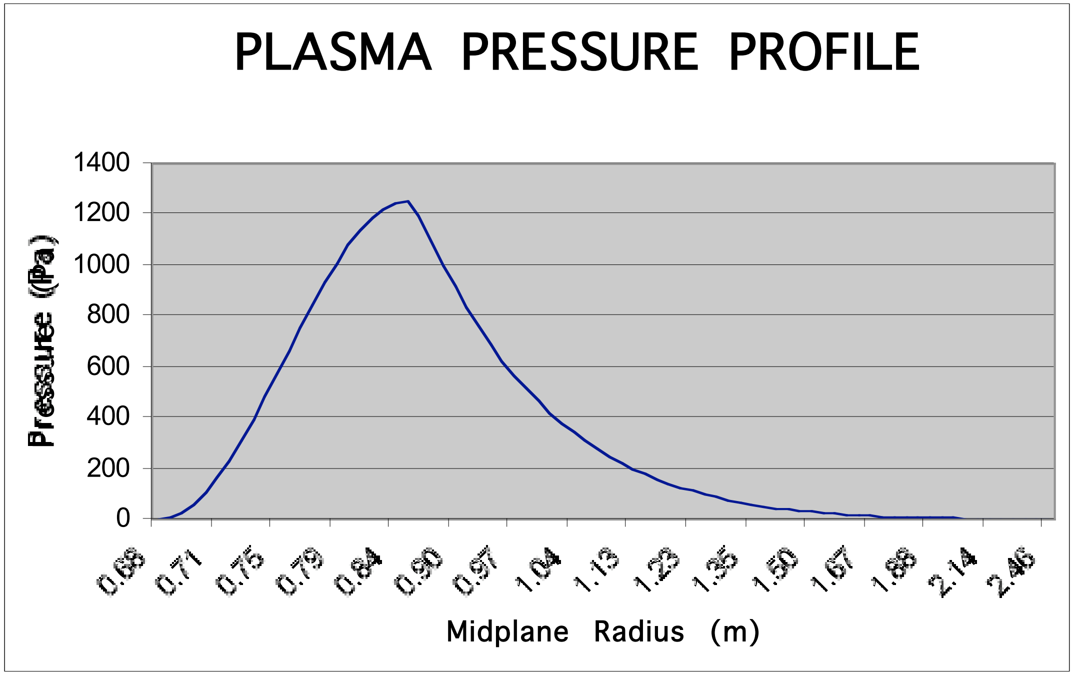
$R_{peak} = 0.71m$

$g = 5/3$

SAMPLE PLASMA SHAPE ALONG WITH ITS PRESSURE PROFILE



$P_{edge} = 1Pa$
$R_{peak} = 0.85m$
$g = 5/3$



SENSOR LOCATION OPTIMIZATION

Fundamental Concept: There is a functional relationship between plasma parameters and external magnetic measurements.

$$\mathbf{m} = \mathbf{F}(\mathbf{p}), \text{ where}$$

m is an m-dimensional vector of different types of measurements at different positions

p is an n-dimensional vector of plasma parameters

F: $\mathbf{R}^n \rightarrow \mathbf{R}^m$ is the response function

OPTIMIZATION STRATEGY

Goal: Find the function **F** that relates plasma parameters to the measurements. Then the components of $\nabla_{\mathbf{p}}\mathbf{F}$ will give the sensitivity of the different types of measurements at different positions to changes in the plasma parameters.

Method: Execute multiple runs of the equilibrium code to get many sets of **m**'s and **p**'s. Linearize **F** about multiple points in the parameter space in order to obtain a set of linearized **F**'s that are valid in a small region in the parameter space. Solve for each of the linearized **F**'s using the **m**'s and **p**'s corresponding to the region where the linearized **F**'s are valid.

EXPANSION OF THE FUNCTION F

Linearization of **F**:

$$\mathbf{m} = \mathbf{F}(\mathbf{p}) \approx \mathbf{F}(\mathbf{p}_0) + (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet (\mathbf{p} - \mathbf{p}_0)$$

$$= \mathbf{F}(\mathbf{p}_0) - (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet \mathbf{p}_0 + (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet \mathbf{p}$$

$$\equiv \mathbf{k} + \mathbf{R} \bullet \mathbf{p}, \text{ where } \quad \mathbf{k} \equiv \mathbf{F}(\mathbf{p}_0) - (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0} \bullet \mathbf{p}_0$$

$$\mathbf{R} \equiv (\nabla_{\mathbf{p}}\mathbf{F})^T|_{\mathbf{p} = \mathbf{p}_0}$$

The problem of finding the function **F** is now reduced to finding the elements of **k** and **R** for each small subdomain of the parameter space.

COUNTING THE UNKNOWNS

- Notice that **k** is an m-element vector and **R** is an $m \times n$ matrix.

- We need $m + m \times n$ independent equations in order to solve for all the elements of **k** and **R**.

- A single set of **m** and **p** gives m independent equations.

> □ We require n+1 sets of **m** and **p**, i.e. ($\mathbf{m}^1, \ldots, \mathbf{m}^{n+1}$), ($\mathbf{p}^1, \ldots, \mathbf{p}^{n+1}$), to obtain the elements of **k** and **R**.

APPLICATION TO LDX

Specific Example (LDX plasma):

The plasma is characterized by the following three parameters:

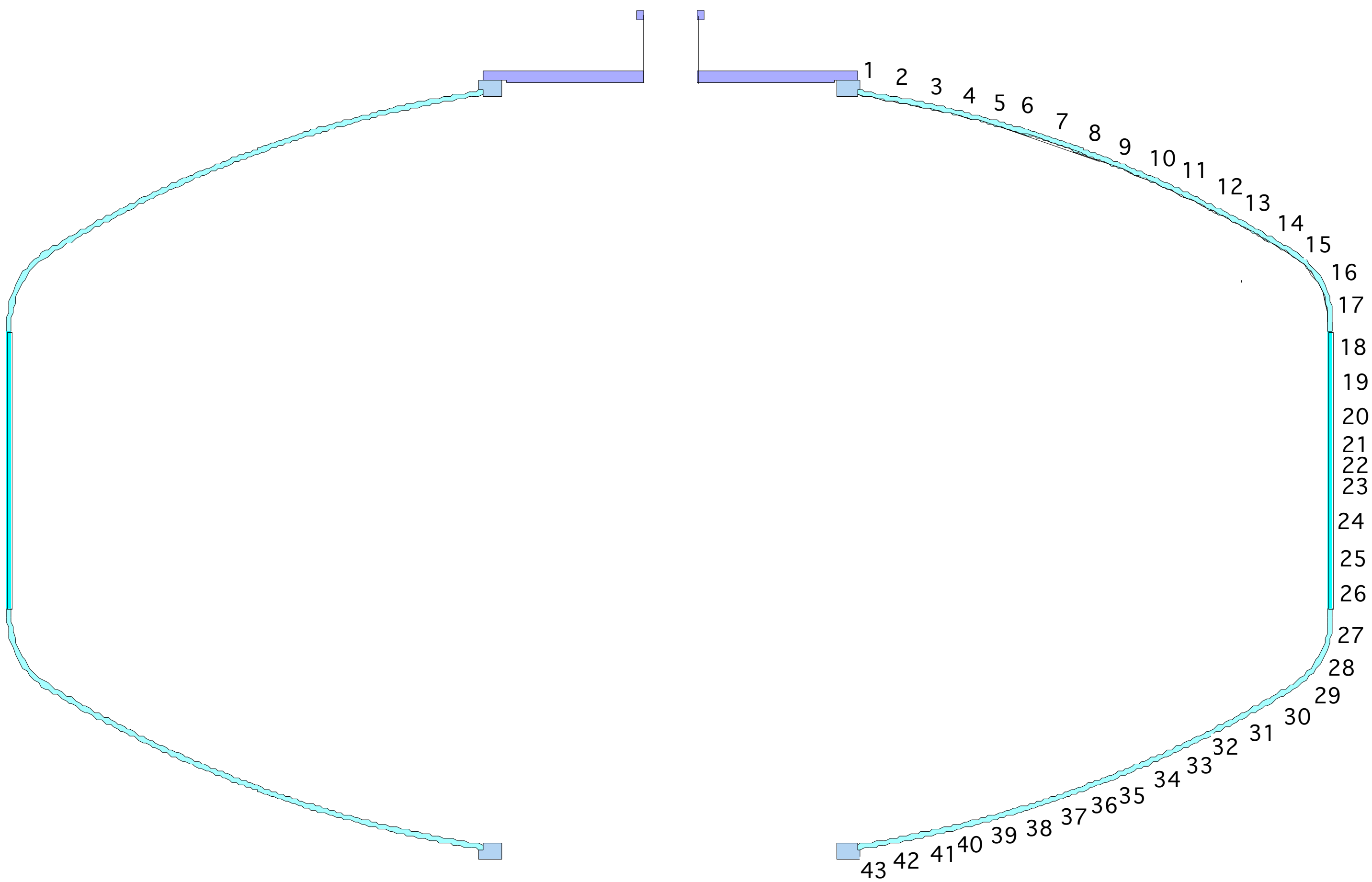
$$\mathbf{p} = [p_1, p_2, p_3] = [p_{edge}, r_{peak}, f_{crit}]$$

We are looking at 3 different types of measurements at 43 possible locations on the vacuum vessel:

$$\mathbf{m} = [m_{BT1},\ldots, m_{BT43}, m_{BN1},\ldots, m_{BN43}, m_{F1},\ldots, m_{F43}]$$

where m_{BTi} = B_p-coil measurement in the tangential direction
m_{BNi} = B_p-coil measurement in the normal direction
m_{Fi} = Flux-loop measurement

VACUUM VESSEL TARGET POINTS



NUMBER OF NECESSARY EQUATIONS

Specific Example (cont'd):

- Notice that m = 129 and n = 3 → 516 unknowns!
- We need 3 + 1 = 4 sets of **m**'s and **p**'s to determine the unknowns
- One of the sets can be used to eliminate **k** and the remaining 3 can be used to solve for **R**

We have:

$$\mathbf{m}^1 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^1$$
$$\mathbf{m}^2 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^2$$
$$\mathbf{m}^3 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^3$$
$$\mathbf{m}^4 = \mathbf{k} + \mathbf{R} \bullet \mathbf{p}^4$$

→

$$(\mathbf{m}^1 - \mathbf{m}^4) = \mathbf{R} \bullet (\mathbf{p}^1 - \mathbf{p}^4)$$
$$(\mathbf{m}^2 - \mathbf{m}^4) = \mathbf{R} \bullet (\mathbf{p}^2 - \mathbf{p}^4)$$
$$(\mathbf{m}^3 - \mathbf{m}^4) = \mathbf{R} \bullet (\mathbf{p}^3 - \mathbf{p}^4)$$
$$\mathbf{k} = \mathbf{m}^4 - \mathbf{R} \bullet \mathbf{p}^4$$

THE RESPONSE MATRIX

Now, concatenate these equations to form a larger system:

$$\begin{pmatrix} \mathbf{m}^1 - \mathbf{m}^4 \\ \mathbf{m}^2 - \mathbf{m}^4 \\ \mathbf{m}^3 - \mathbf{m}^4 \end{pmatrix} = \begin{pmatrix} \mathbf{R} & & \\ & \mathbf{R} & \\ & & \mathbf{R} \end{pmatrix} \bullet \begin{pmatrix} \mathbf{p}^1 - \mathbf{p}^4 \\ \mathbf{p}^2 - \mathbf{p}^4 \\ \mathbf{p}^3 - \mathbf{p}^4 \end{pmatrix}$$

We can solve this system for **R**.

Notice:

- $\mathbf{R} = (\square_{\mathbf{p}}\mathbf{F})^T$ gives a measure of sensitivity.
- **R** is valid only for ranges in the vicinity of $\mathbf{p}^1$, $\mathbf{p}^2$, $\mathbf{p}^3$, and $\mathbf{p}^4$.

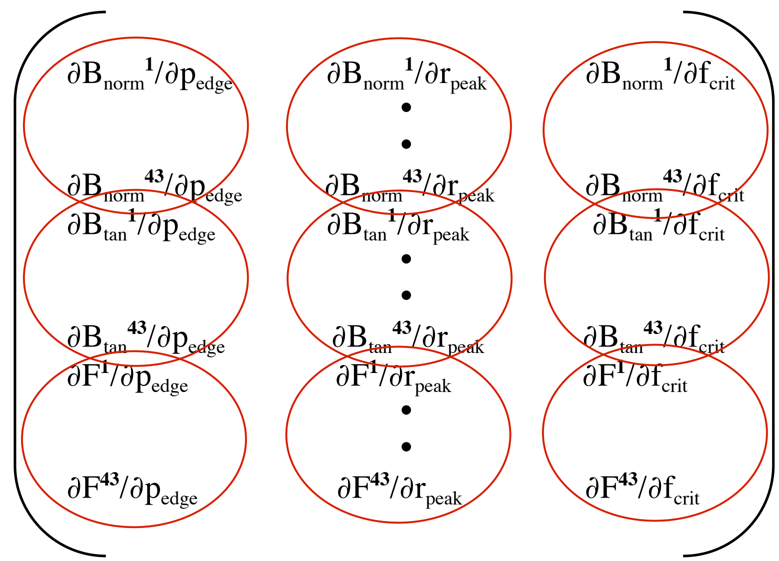
EXPLICIT STRUCTURE OF R

$$
\mathbf{R} = \begin{pmatrix}
\partial B_{norm}^{1}/\partial p_{edge} & \partial B_{norm}^{1}/\partial r_{peak} & \partial B_{norm}^{1}/\partial f_{crit} \\
 & \vdots & \\
\partial B_{norm}^{43}/\partial p_{edge} & \partial B_{norm}^{43}/\partial r_{peak} & \partial B_{norm}^{43}/\partial f_{crit} \\
\partial B_{tan}^{1}/\partial p_{edge} & \partial B_{tan}^{1}/\partial r_{peak} & \partial B_{tan}^{1}/\partial f_{crit} \\
 & \vdots & \\
\partial B_{tan}^{43}/\partial p_{edge} & \partial B_{tan}^{43}/\partial r_{peak} & \partial B_{tan}^{43}/\partial f_{crit} \\
\partial F^{1}/\partial p_{edge} & \partial F^{1}/\partial r_{peak} & \partial F^{1}/\partial f_{crit} \\
 & \vdots & \\
\partial F^{43}/\partial p_{edge} & \partial F^{43}/\partial r_{peak} & \partial F^{43}/\partial f_{crit}
\end{pmatrix}
$$

Because we have many **R**’s corresponding to different points in the parameter space, we need to have a method of picking out the most sensitive positions averaged over the whole space.

CHOOSING THE OPTIMAL POSITIONS

For each **R**, we can divide the matrix into 9 parts as shown on the right. For each part, tabulate the number of times each position comes within the 9 highest sensitivities per **R** over all the **R**'s. The resulting histograms will elucidate the optimal positions for each sensor type.



EXECUTION OF THE EQUILIBRIUM CODE

- 1,328 successful runs have been executed over the following parameter ranges:

  $0.1\ \text{Pa} < P_{edge} < 10.0\ \text{Pa}$

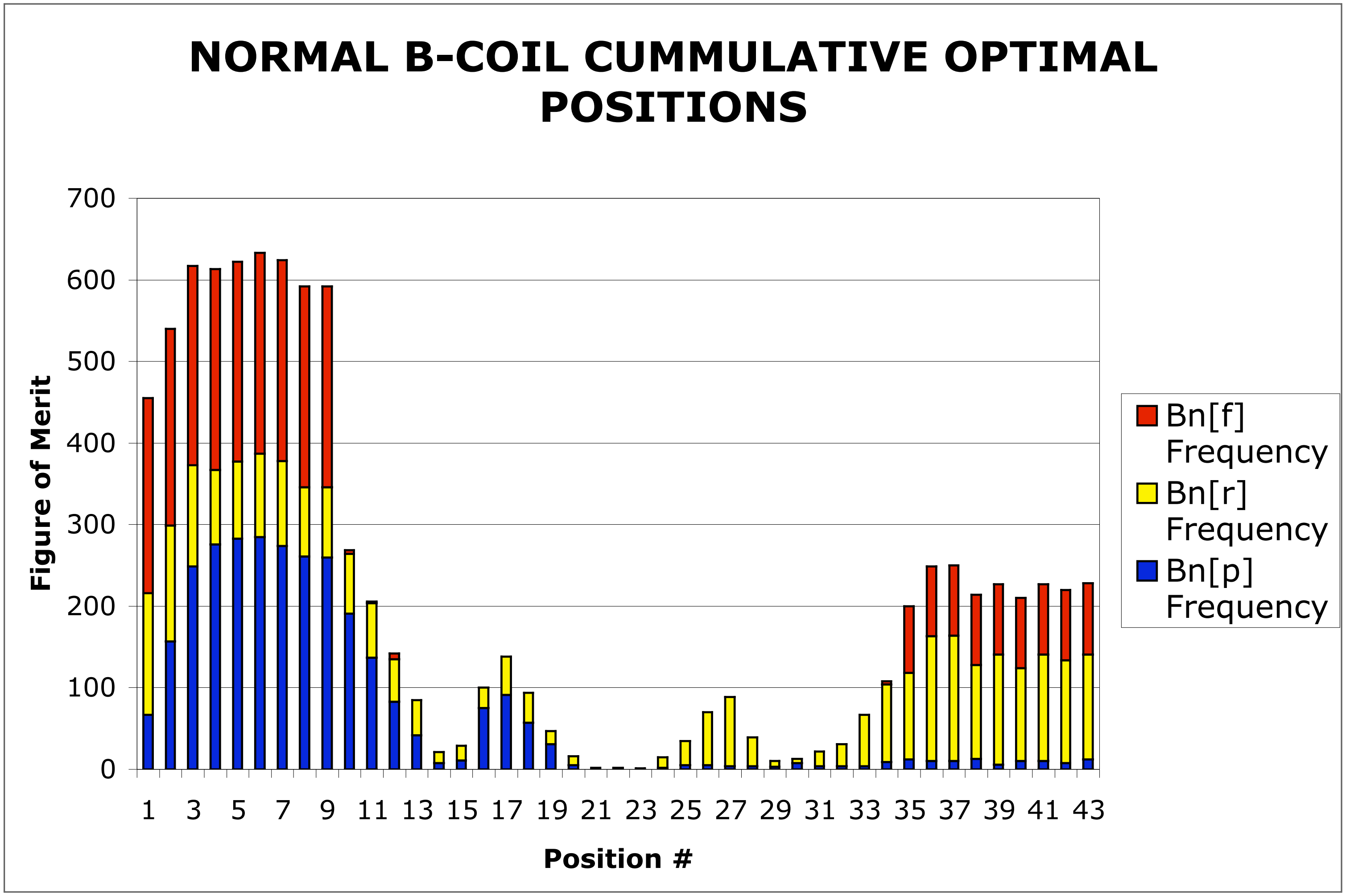
  $0.50\ \text{m} < R_{peak} < 0.90\ \text{m}$

  $0.5 < F_{crit} < 3.0$ (where □= 5/3 □ F_{crit})

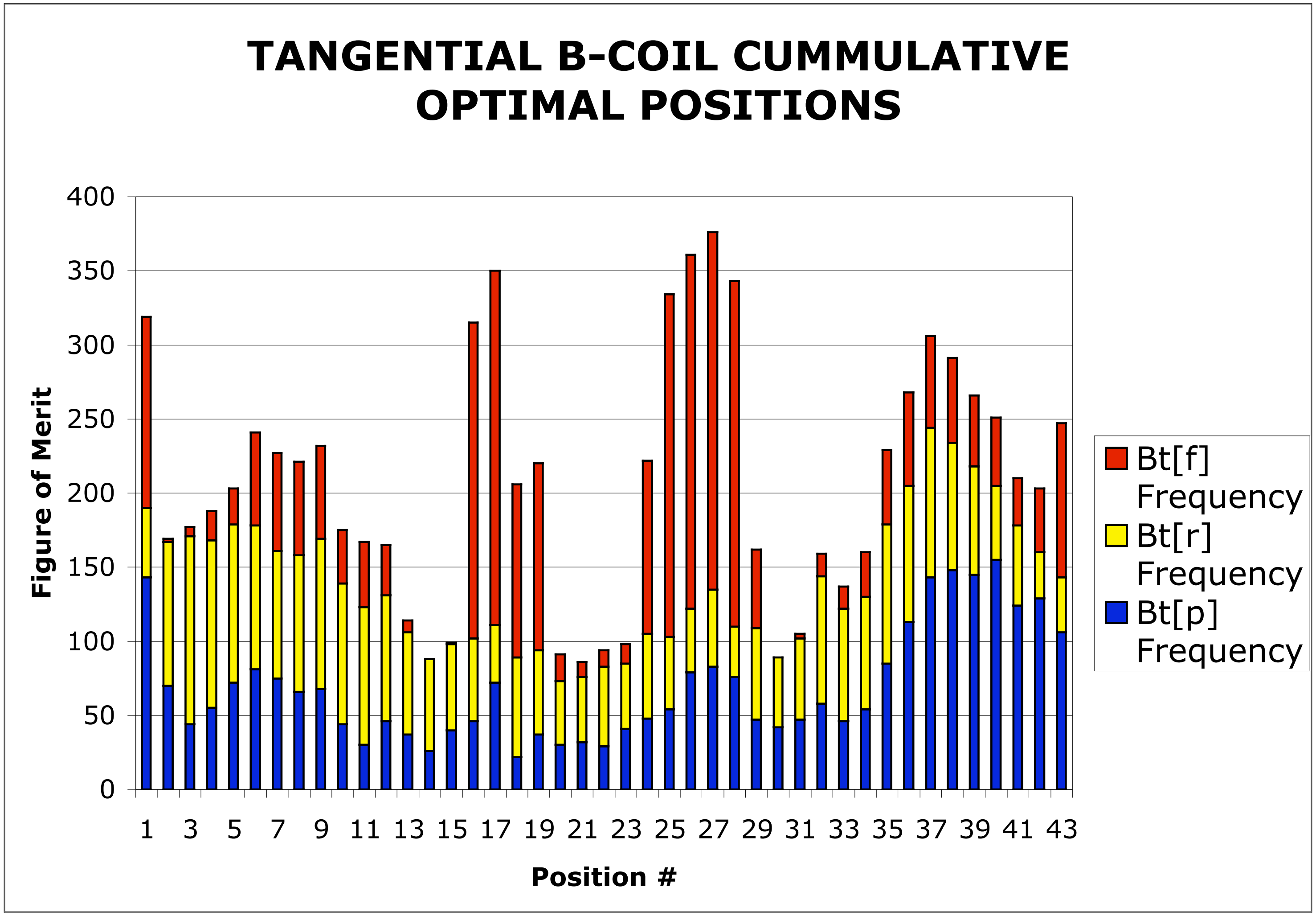
- The code failed to converge properly for $F_{crit} > 1.5$ due to the excessively high plasma pressure attained during the code run.

- The successful runs yielded a total of 332 response matrices.

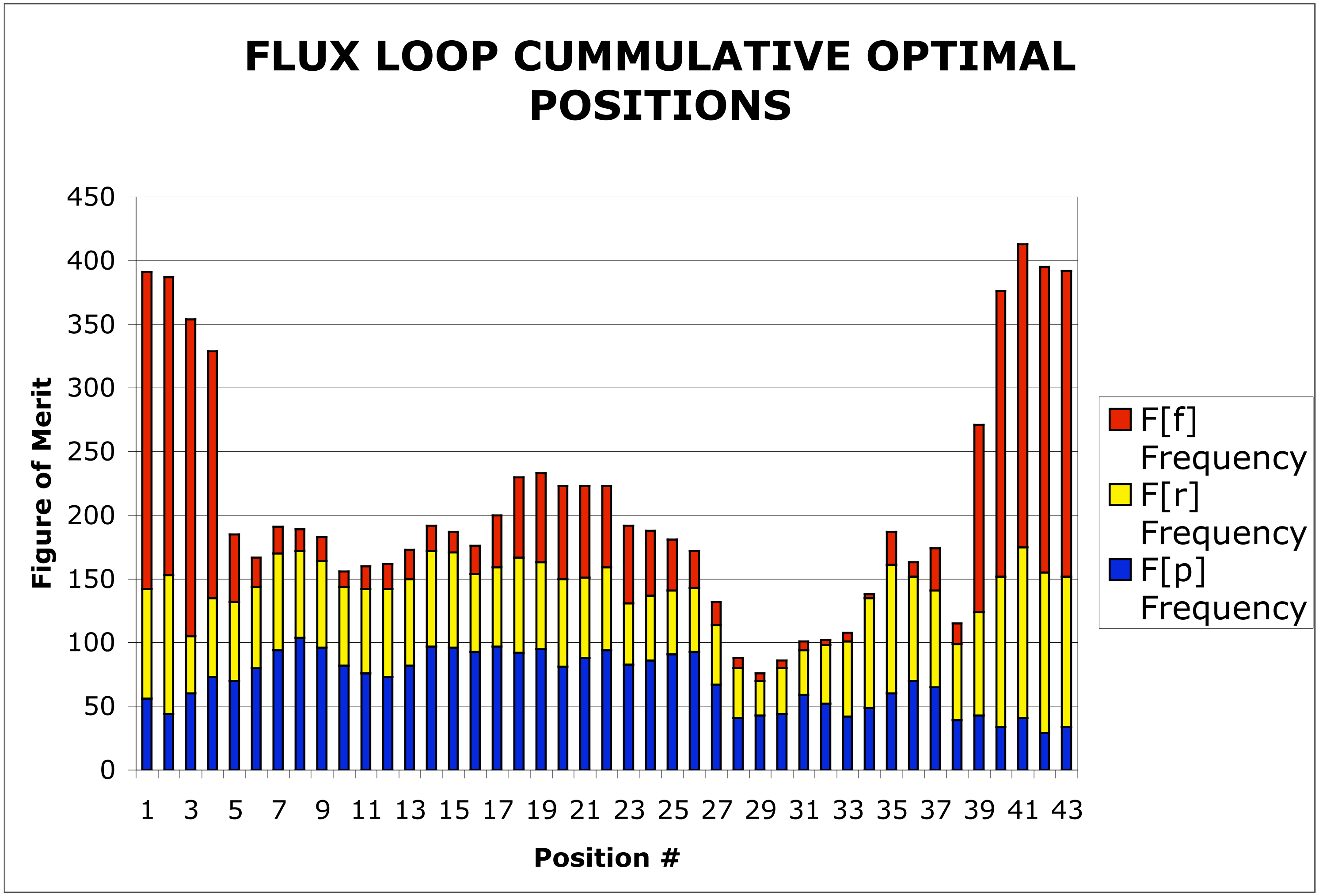
OPTIMIZATION RESULTS



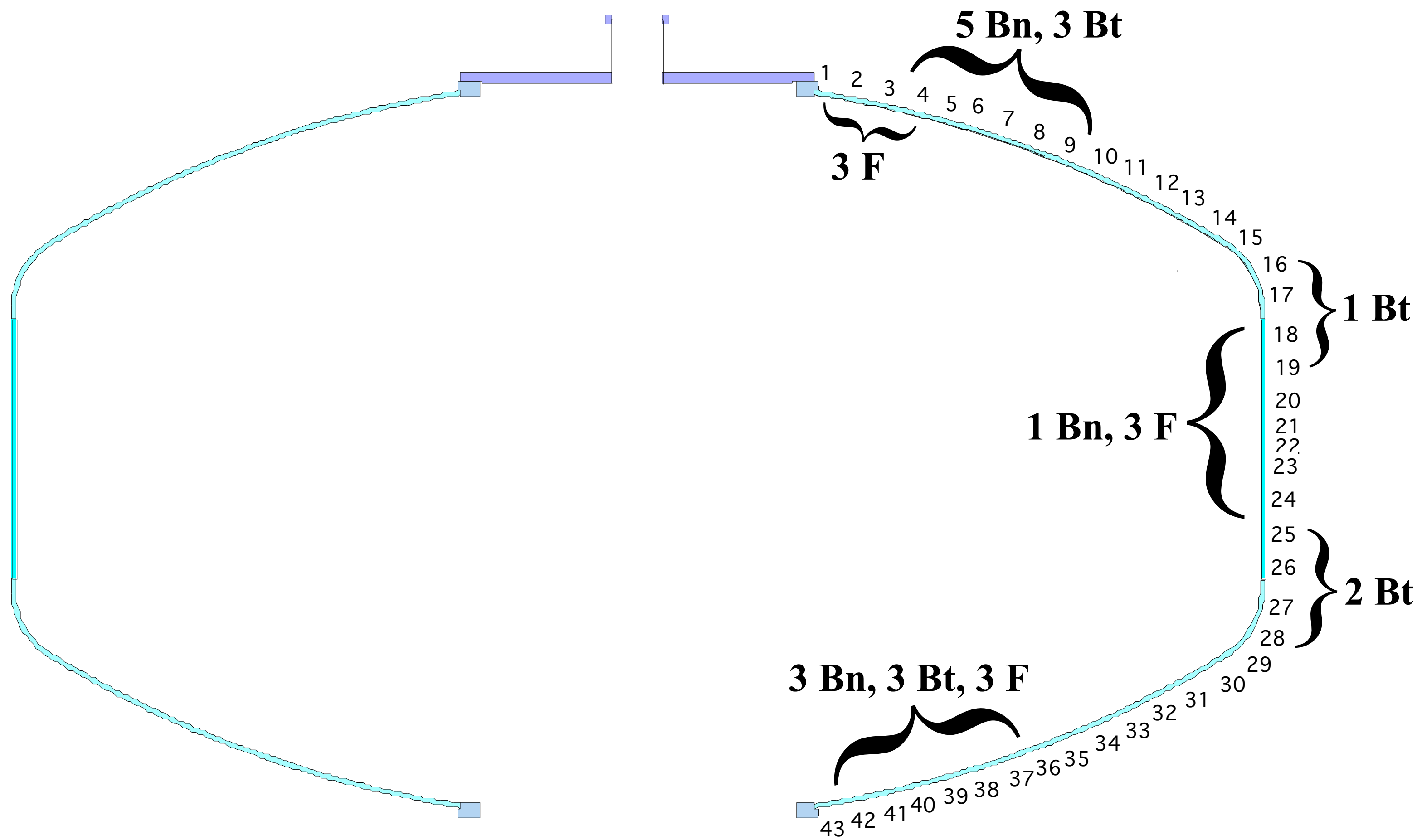
OPTIMIZATION RESULTS



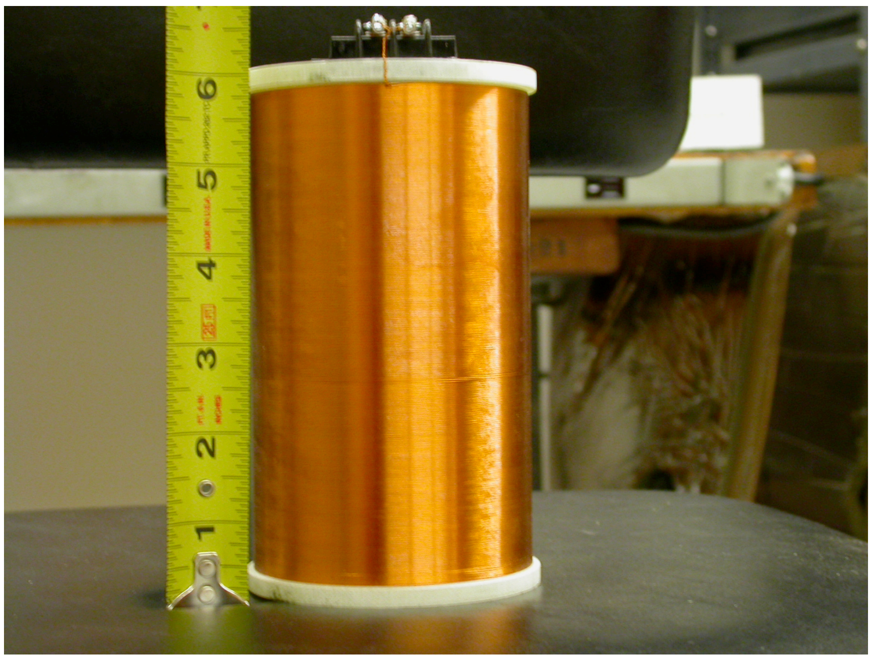
OPTIMIZATION RESULTS

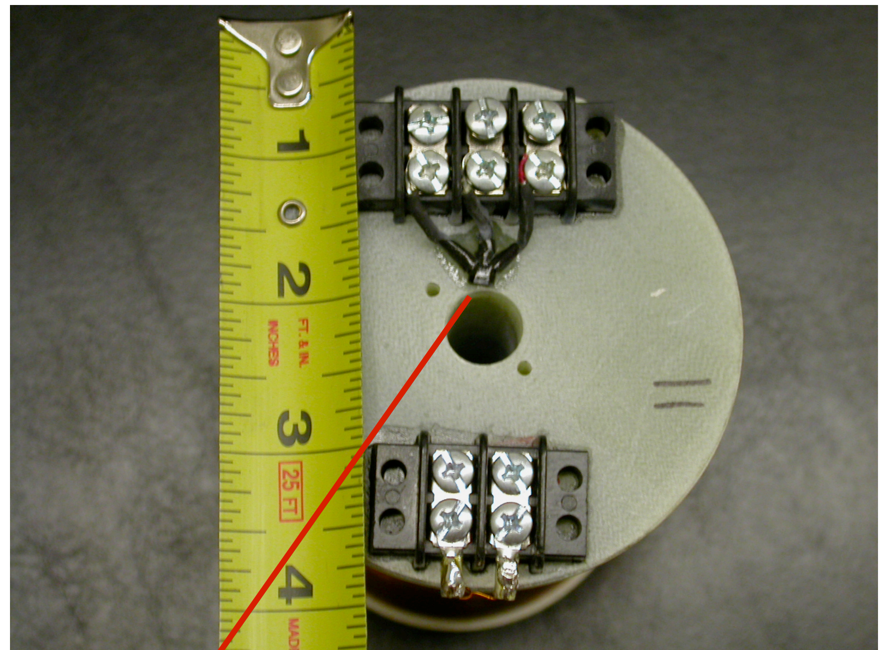


OPTIMIZATION RESULTS



LDX B_P-COIL AND HALL SENSOR





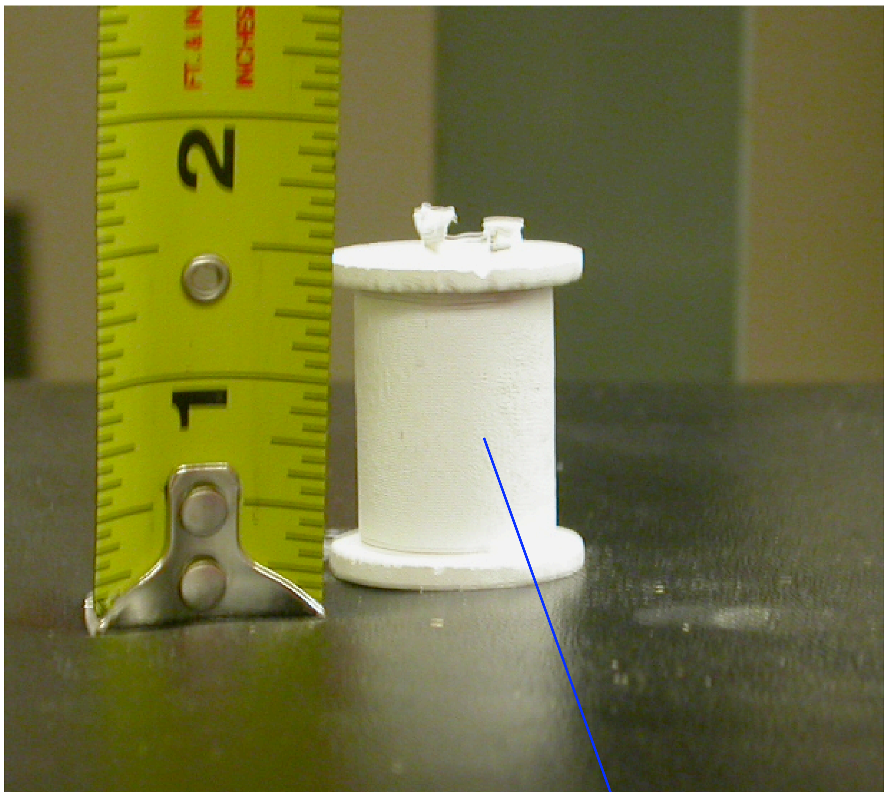
B_p-Coil Specs:

- NA ~ 5 m^2
- Sensitivity: 500 mV/G
  (connected to a 1 ms RC integrator)

Hall Sensor Specs:

- Field Range: +/- 500 G
- Sensitivity: 5 mV/G

LDX MIRNOV COIL



Boron Nitride Coated

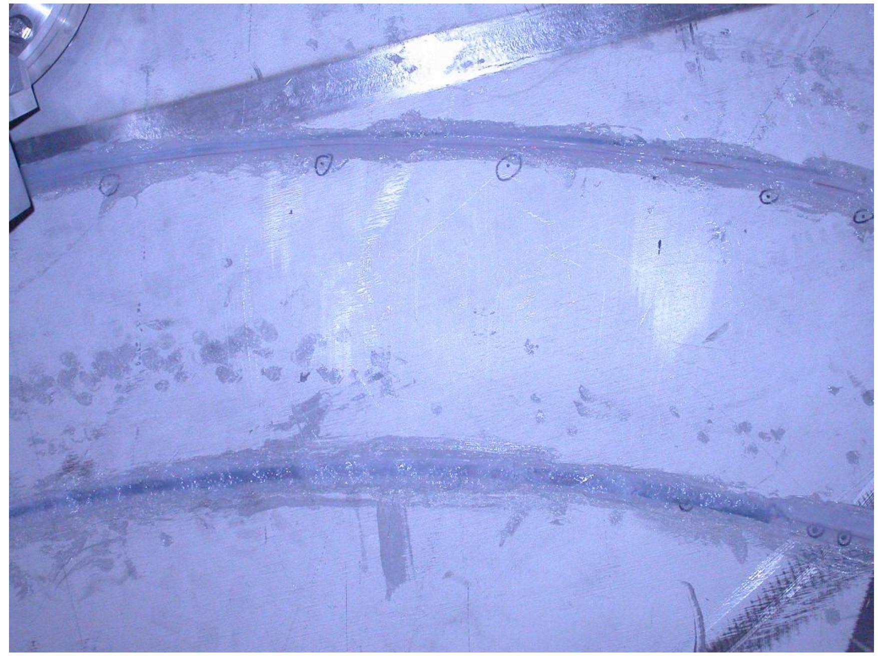
NA: $\sim 0.06\ m^2$

L/R_0: ~ 50 ps

f_0: ~20 GHz

- Directly measures dB/dt
- Placed inside the vessel
- Measures fluctuations in the microsecond range

LDX FLUX LOOPS



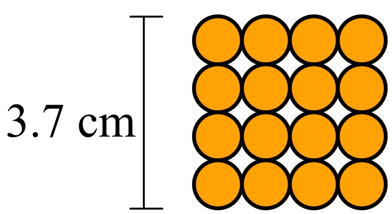
Close-up view

22 AWG magnet wire sealed by silicone caulk

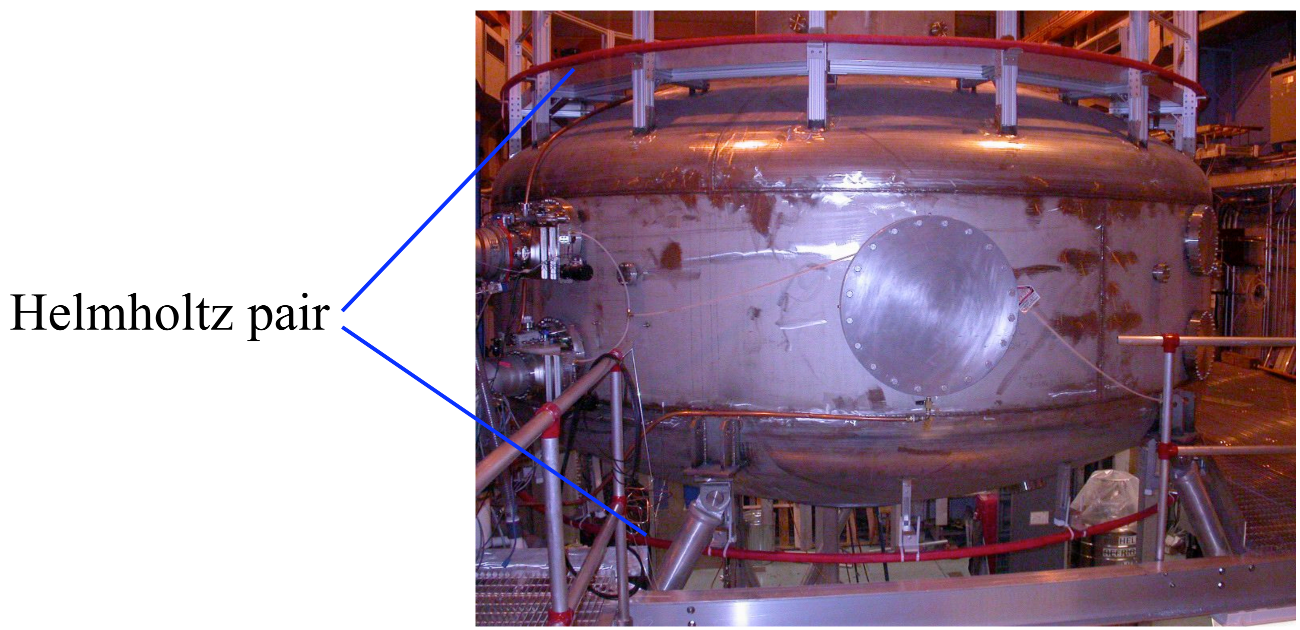
(connected to a 1 ms RC integrator)

LDX HELMHOLTZ COILS

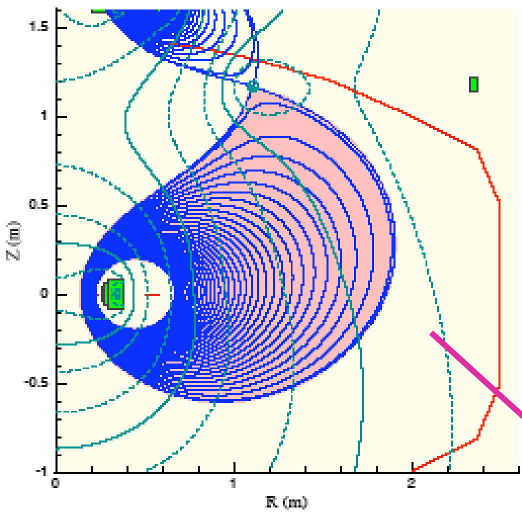
- Will be used to calibrate magnetic diagnostics and manipulate plasma shape.
- R = 244 cm, N = 16 turns of 00 AWG copper wire.
- Designed to carry 80 kA-turns for ≤ 1 sec.

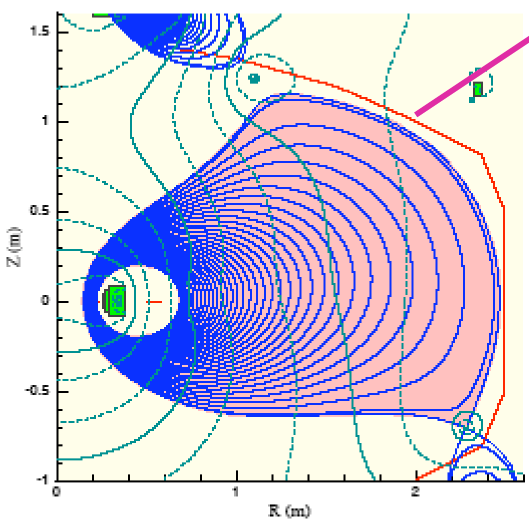


Coil Cross-section

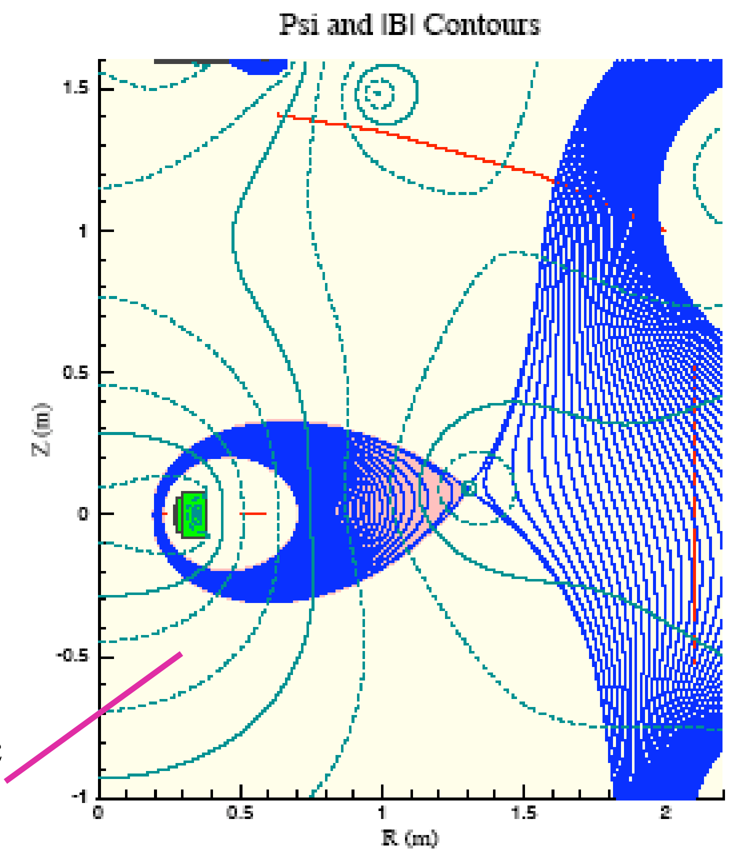


HELMHOLTZ COIL MANIPULATED EQUILIBRIA





Up-Down Asymmetric Currents



Symmetric Currents