Stability of Axisymmetric Plasmas in Closed Line Magnetic Fields

A. N. Simakov 1), H. Vernon Wong 2), H. L. Berk 2), P. J. Catto 1), C. Crabtree 2), R. J. Hastie 1), 3), W. Horton 2), J. Kesner 1), M. Kotschenreuther 2), J. J. Ramos 1), J. W. Van Dam 2)

1) MIT Plasma Science and Fusion Center, Cambridge, Massachusetts, U.S.A.
2) Institute for Fusion Studies, University of Texas at Austin, Austin, Texas, U.S.A.
3) EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, Oxfordshire, U.K.

e-mail contact of main author: simakov@mit.edu

Abstract. The stability of axisymmetric plasmas confined by closed poloidal magnetic field lines is considered. The results are relevant to plasmas in the dipolar fields of stars and planets, as well as the Levitated Dipole Experiment, multipoles, Z pinches and field reversed configurations. The ideal MHD energy principle is employed to study the stability of pressure driven shear Alfvén modes. A point dipole is considered in detail to demonstrate that equilibria exist which are MHD stable for arbitrary beta. Effects of sound waves and plasma resistivity are investigated for Z pinch and point dipole equilibria by means of resistive MHD theory. Kinetic theory is used to study drift frequency modes and their interaction with MHD modes near the ideal stability boundary for different collisionality regimes. Effects of collisional dissipation on drift mode stability are explicitly evaluated and applied to a Z pinch. The role of finite Larmor radius effects and drift reversed particles in modifying ideal stability thresholds is examined.

1. Introduction

Stability of axisymmetric plasmas confined by closed line poloidal magnetic fields is investigated. The results are relevant to both natural systems, such as plasmas confined by dipolar fields of stars and planets, and laboratory experiments, such as the Levitated Dipole Experiment (LDX) [1], multipoles, Z pinches and field reversed configurations. In such systems, plasma and magnetic field compression due to closed field lines or large trapped particle populations counteracts unfavorable magnetic field line curvature, providing favorable stability properties.

2. Ideal MHD Stability

When ideal MHD theory is employed to study isotropic pressure plasmas, the energy principle can be used to derive an interchange stability condition, $d < \gamma$, and an integro-differential ballooning equation,

$$\boldsymbol{B}\cdot\boldsymbol{\nabla}\left(\frac{\boldsymbol{B}\cdot\boldsymbol{\nabla}\xi_\psi}{R^2B^2}\right)+\frac{4\pi}{R^2B^2}\left(2\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}p+\rho\,\omega^2\right)\xi_\psi=16\pi\gamma p\left(\frac{\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}\psi}{R^2B^2}\right)\frac{\langle\xi_\psi\,\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}\psi/R^2B^2\rangle_\theta}{1+4\pi\gamma p\,\langle B^{-2}\rangle_\theta},\tag{1}$$

for pressure driven shear Alfvén modes in the most unstable limit of large toroidal mode numbers, $n \gg 1$. Here $d \equiv -\mathrm{d}\ln p/\mathrm{d}\ln\left(\oint \mathrm{d}\ell/B\right)$, $\gamma = 5/3$, R is the cylindrical radial coordinate, $\boldsymbol{\kappa}$ is the magnetic curvature, ρ is plasma density, ξ_ψ describes the radial plasma displacement, and $\langle\cdots\rangle_\theta$ and $\mathrm{d}\ell$ indicate an average and the incremental length along the field line. The results are applied to a point dipole equilibrium [2] to show that it is stable for arbitrary $\beta \equiv 8\pi p/B^2$.

3. Resistive MHD Stability

The ideal MHD treatment can be generalized to include sound waves and effects of plasma resistivity. For $n \gg 1$ the system of linearized resistive MHD equations reduces [3] to

$$\boldsymbol{B}\cdot\boldsymbol{\nabla}\left[\frac{\boldsymbol{B}\cdot\boldsymbol{\nabla}\xi_\psi}{R^2B^2\left(1+\mathrm{i}c^2n^2\eta_\parallel/4\pi R^2\omega\right)}\right]+\frac{4\pi}{R^2B^2}\left(2\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}p+\rho\,\omega^2\right)\xi_\psi+2\left(\frac{\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}\psi}{R^2B^2}\right)W=0,$$

$$\boldsymbol{B}\cdot\boldsymbol{\nabla}\left(\frac{\boldsymbol{B}\cdot\boldsymbol{\nabla}W}{B^2}\right)+4\pi\rho\omega^2\left[2\left(\frac{\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}\psi}{R^2B^2}\right)+\frac{\mathrm{i}c^2n^2\left(\eta_\perp-\eta_\parallel\right)}{R^2B^2\omega}\frac{\mathrm{d}p}{\mathrm{d}\psi}\right]\xi_\psi+ \qquad (2)$$

$$\frac{\rho\omega^2}{\gamma p}\left[1+\frac{4\pi\gamma p}{B^2}\left(1+\frac{\mathrm{i}c^2n^2\eta_\perp}{4\pi R^2\omega}\right)\right]W-2\frac{\mathrm{i}c^2n^2\eta_\parallel}{\omega}\left(\frac{\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}p}{R^2B^2}\right)\left(4\pi\frac{\mathrm{d}p}{\mathrm{d}\psi}\xi_\psi+W\right)=0,$$

where $W \equiv 4\pi\gamma p\,(\boldsymbol{\nabla}\cdot\boldsymbol{\xi})$, $\eta_\parallel$ ($\eta_\perp$) is parallel (perpendicular) resistivity, and c is the speed of light. In the limit $\omega \to 0$ Eqs. (2) reduce to Eq. (1). Concentrating on up-down symmetric systems we find from Eqs. (2) that two types of resistive instabilities are possible: 'strong' and 'weak' ones. The 'strong' instabilities are purely growing modes with frequency $\omega = \mathrm{i}\left(c^2n^2\eta_\parallel/\rho F\right)^{1/3}(\mathrm{d}p/\mathrm{d}\psi)^{2/3} \propto \eta_\parallel^{1/3}$, where $F \propto \langle(\boldsymbol{B}\cdot\boldsymbol{\nabla}\xi_{\psi 0})/R^2\rangle_\theta^{-2}$ is a positive number [3], with $\xi_{\psi 0}$ a solution of Eq. (1) with inertia and plasma plus magnetic field compression terms set to zero. These instabilities only exist at the ideal stability boundary for up-down anti-symmetric (odd) modes and do not rely on a radial localization, unlike resistive modes in sheared magnetic fields. The quantity $F \to \infty$ for a circular Z pinch, so the 'strong' instability is of no concern there. The 'weak' instabilities have real frequencies associated with ideally stable sound or shear Alfvén modes, and growth rates proportional to $n^2\eta_{\parallel,\perp}$, which are $n^2 \gg 1$ times larger that the inverse equilibrium resistive evolution time scale. Such instabilities can exist away from odd mode ideal stability boundaries. In a circular Z pinch, the 'weak' instabilities are of no concern as well since they are more stable than the ideal interchange mode. In the point dipole [2], where the lowest ideal odd mode is always stable so that the 'strong' resistive instability is not allowed, this lowest ideal odd mode is destabilized by $\eta_\parallel$ and acquires a growth rate proportional to $n^2\eta_\parallel$, unless $\eta_\parallel \ll \eta_\perp$.

4. Kinetic Stability

MHD fails to describe many important phenomena, such as drift frequency modes, and assumes collisional orderings which are often not relevant. In particular, for the anticipated typical LDX plasmas with densities $N \sim 5\times(10^{12}-10^{13})$ cm^{-3} and temperatures $T_e \sim T_i \sim (100-200)$ eV electrons are 'semi-collisional', $\omega_{be} > \nu_e > \omega_{de}$, ω_{*e}, while ions can be anything between 'semi-collisional' and collisionless. Here, ω_b, ν, ω_d and ω_* are bounce, collision, magnetic and diamagnetic drift frequencies, respectively. Accordingly, a kinetic theory has been employed to study the stability of plasmas with i) both 'semi-collisional' electrons and ions, ii) both collisionless electrons and ions, and finally iii) 'semi-collisional' electrons and collisionless ions.

To study the stability of 'semi-collisional' plasmas i) at arbitrary β the full linearized gyro-kinetic equation is solved perturbatively [4]. The leading order correction to the distribution function is found to be a modified stationary Maxwellian, which, upon substitution into quasineutrality condition and Ampere's law, leads to a ballooning equation, which is equivalent to Eq. (1), but with $\gamma \to \Gamma = \Gamma\left(\omega,\omega_{di},\omega_{*i}\right)$. For $\omega \gg \omega_{di}$, ω_{*i}, Eq. (1)

is recovered, since $\Gamma \to 5/3$, while for $\omega \sim \omega_{di}$, ω_{*i} the dispersion relation for the so-called entropy mode [5] is obtained,

$$\left(d - \frac{5}{3}\right)\omega^2 + \frac{5}{9}\left(d\,\frac{3\eta - 7}{1+\eta} + 5\right)\langle\omega_{di}\rangle_\theta^2 = 0, \tag{3}$$

where $\eta \equiv \mathrm{d}\ln T_i/\mathrm{d}\ln N$ and $T_e = T_i$ is assumed. The stability of this mode depends on both d and η, as shown in FIG. 1. When stable, the entropy mode is represented by two waves propagating in the ion and the electron diamagnetic drift directions, respectively, with the electrons and ions oscillating radially and the total plasma pressure being unperturbed. When unstable, convective instability tries to relax the 'unfavorable' temperature and density gradients. The entropy mode is electrostatic and flute-like at arbitrary β. As the shear Alfvén mode becomes flute-like in the vicinity of its marginal stability boundary, $d = 5/3$, it is not surprising that the two modes couple at such d. Ion collisional effects, in particular 'gyro-relaxation effects', corresponding to collisional thermal redistribution among the different degrees of freedom and described by the ion viscosity, play an important role in the stability of the entropy mode and have been found to be capable of destabilizing it (FIG. 1).

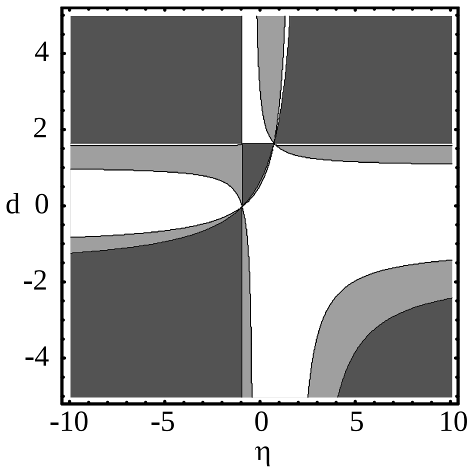

FIG. 1. Regions of stability (white) and instability in the absence (black) and due to (gray) gyro-relaxation effects for the entropy mode for a Z pinch equilibrium.

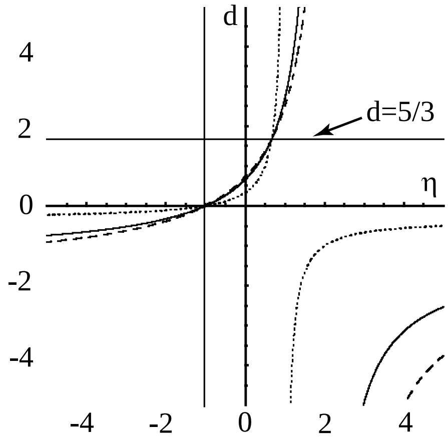

FIG. 2. Stability boundaries for the entropy mode for the collisionality regimes i) (dashed), ii) (dotted) and iii) (solid).

The cases ii) of collisionless plasma and iii) of 'semi-collisional' electrons and collisionless ions have been studied in a similar way [6] in the limit of small β. The analogs of the shear Alfvén and the entropy modes are found, with the former being stable (unstable) when $d < 5/3$ ($d > 5/3$) while the stability of the latter depending on d and η with the most favorable for stability value of η being 2/3. It is found that the instability regions of the drift-frequency mode in $d - \eta$ space become larger as the collisionality decreases, being the largest for the collisionless case (FIG. 2).

5. FLR Stabilization of High-β Collisionless Plasma

The case (ii) of collisionless plasma has also been studied for $\beta \gg 1$ in Ref. [7] in an attempt to examine the stability of the Earth's magnetospheric magnetic flux surfaces and to suggest pressure-driven plasma instabilities as a possible trigger for magnetic substorms. A quadratic form of eigenmode equations for electromagnetic perturbations derived from a collisionless drift kinetic equation is used. Sufficient conditions for the collisionless Alfvén and entropy mode stability are formulated in terms of η and a d-like quantity depending on the ion pitch angle variable, the former condition being identical to the usual MHD stability condition, $d < 5/3$. The local stability of a model dipolar plasma equilibrium has been investigated to find that, unlike the point dipole equilibrium [2], it is MHD unstable above some critical β and that the most favorable value of η for the entropy mode stability is 2/3, as in Ref. [6]. It is of interest to note that the Earth's high-pressure trapped geotail plasma satisfies MHD ballooning stability condition except during magnetic substorms [8].

At the outer flux surfaces of finite volume magnetic dipole configurations where the pressure may be forced to decrease over a scale length, L_p, shorter than the field line curvature, the criterion for compressional stabilization, $d < 5/3$, is violated and collisionless MHD instabilities are likely to occur. Here we address whether it is possible to reduce their growth rates by finite ion Larmor radius (r_L) effects. To explore this possibility, the stability of the outer boundary of a hard-core cylindrically symmetric kinetic Z-pinch equilibrium is investigated. The integral eigenmode equation for arbitrary r_L is derived and solved numerically for flute perturbations. Ignoring compressional stabilizing terms of order $\kappa^2 = R^{-2}$, assuming $\partial/\partial R > k_z$, and considering the small r_L limit, the integral equation reduces to a differential equation (see also Ref. [9])

$$\left(\frac{\partial}{\partial R} - \frac{4\pi}{B}\frac{\partial p}{\partial\tilde{\psi}}\right)\left\{R\mathcal{T}\left(\frac{\partial\xi_\psi}{\partial R} + \frac{4\pi}{B}\frac{\partial p}{\partial\tilde{\psi}}\xi_\psi\right)\right\} + 2\frac{k_z^2}{B}\frac{\partial p}{\partial\tilde{\psi}}\,\xi_\psi = 0, \tag{4}$$

where $\mathcal{T} \equiv (\rho/B^2)\left\{(\omega-\omega_E)^2 - (\omega-\omega_E)\,\omega_{*p}\,(1-\beta/2) - \omega_{*p}\,\omega_g\right\}$ contains the FLR corrections, $\omega_E = k_z c\,(\partial\phi_0/\partial\tilde{\psi})$, $\omega_{*p} = (k_z c/eN)\,(\partial p/\partial\tilde{\psi})$, $\omega_g = (\pi k_z c/2eB^2) \times (\partial/\partial\tilde{\psi})\int \mathrm{d}^3v\, F_0 m^2 v_\perp^4$; and ϕ_0, $\boldsymbol{B} = \boldsymbol{\nabla}\tilde{\psi}\times\boldsymbol{\nabla}z$, F_0 and m are the equilibrium electrostatic potential, magnetic field, ion distribution function and ion mass, respectively.

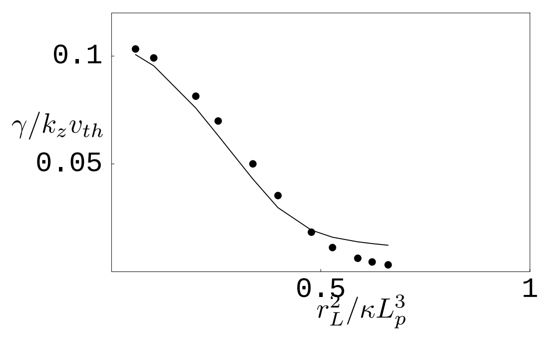

FIG. 3. Growth rates for hard-core Z-pinch equilibrium with conducting wall at plasma boundary: from integral equation (solid curve); from differential equation (dots).

We find from solutions of both the integral and differential equations that a critical requirement for FLR to be effective in reducing MHD growth rates is to have a conducting wall at the plasma boundary, which constrains the boundary perturbation to be zero, $\xi_\psi = 0$. If the density is finite at the plasma boundary, $\mathcal{T}$ is finite and the differential equation is nonsingular at the plasma boundary. Then, when the pressure gradient is finite, FLR effects reduce MHD growth rates if $r_L^2 \sim \kappa L_p^3$ (FIG. 3). The solutions of the integral eigenmode equation are also shown in FIG. 3 and they are similar to those of the differential eigenmode equation. However, in practice a vacuum gap and a scrape-off region is present. With a vacuum gap, and when the plasma frequency exceeds the cyclotron frequency ($\omega_{pi}/\Omega_i \gg 1$) within the unstable boundary layer, the boundary condition effectively changes to $\partial\xi_\psi/\partial R \to 0$. We then find that FLR stabilization is not effective if the conducting wall is at a distance larger than a small fraction of a Larmor radius from the plasma edge (this result applies to both the integral and differential equations).

At high plasma beta, $\beta > \kappa L_p$, particle drifts are reversed. If magnetic field gradient drifts are larger than typical MHD growth rates, the magnetic compression dominates the plasma perturbation. The compressional plasma response is now more complicated, but the plasma boundary conditions affect the solutions of the eigenmode equations in a manner similar to that described above. The compressional response is proportional to the pressure gradient, and it can be stabilizing if the curvature drift frequency is also larger than typical MHD growth rates. This requirement is not easy to satisfy although it may be achievable if characteristic MHD growth rates can be reduced (for example) by having a two component plasma in which a small population of 'hot' particles support the pressure gradient while the principal 'background' component provides a large plasma inertia.

6. Conclusions

Ideal MHD is used to demonstrate that axisymmetric closed field line equilibria exist which are stable for arbitrary β. Resistive MHD is employed next to show that two types of resistive instabilities are possible in such systems and to evaluate their growth rates. Kinetic theory is used to study drift frequency modes and their interaction with MHD modes for different collisionality regimes. It is found that the instability regions of the drift-frequency modes in $d - \eta$ space increase as the collisionality decreases. FLR stabilization of collisionless MHD edge-modes is investigated at $\beta \sim 1$. FLR effects can reduce MHD growth rates only if a conducting wall is placed almost exactly at the plasma boundary. With a vacuum gap and $\omega_{pi}/\Omega_i \gg 1$ (the typical case), it is quite difficult to obtain FLR stabilization, an issue that needs further study.

[1] KESNER, J., BROMBERG, L., MAUEL, M.E., GARNIER, D., "Plasma Confinement in a Magnetic Dipole", Fusion Energy 1998 (Proc. 17th Int. Conf. Yokahama, 1998), CD-ROM file EX4/2 and http://www.iaea.org/programmes/ripc/physics/fec1998/html/fec1998.htm.
[2] KRASHENINNIKOV, S.I., CATTO, P.J., HAZELTINE, R.D., Phys. Rev. Lett. **82** (1999) 2689.
[3] SIMAKOV, A.N., CATTO, P.J., RAMOS, J.J., HASTIE, R.J., to appear in Phys. Plasmas.
[4] SIMAKOV, A.N., HASTIE, R.J., CATTO, P.J., Phys. Plasmas **9** (2002) 201.
[5] KADOMTSEV, B.B., Sov. Phys. - JETP **10** (1960) 780.
[6] KESNER, J., HASTIE, R.J., Phys. Plasmas **9** (2002) 395.
[7] WONG, H.V., HORTON, W., VAN DAM, J.W., CRABTREE, C., Phys. Plasmas **8** (2001) 2415.
[8] HORTON, W., WONG, H.V., VAN DAM, J.W., CRABTREE, C., J. Geophys. Res. **106** (2001) 18803.
[9] PEARLSTEIN, L.D., KRALL, N.A., Phys. Fluids **9** (1966) 2231.

Work supported by U.S. Department of Energy.